

$f(x) = a \quad d: y = ax + 1 \quad f'(x) = a \quad 2a + 1 = a \rightarrow a = \frac{a}{1} \quad f'(x) = \frac{a}{1}$ (1)

$a = \frac{2-1}{2-(-1)} = \frac{1}{3} \quad d: y = \frac{1}{3}x + \frac{4}{3} \quad \frac{1}{3}x + \frac{4}{3} = \sqrt{ax-1} \quad \frac{1}{9}x^2 + \frac{8}{9}x + \frac{16}{9} = ax - 1 \xrightarrow{\times 9} x^2 + 8x + 16 = 9ax - 9$
 $x^2 + (1-9a)x + 25 = 0 \quad \Delta = 0 \rightarrow \begin{cases} a = -\frac{2}{9} \rightarrow \text{غیر قابل رادیکال را منفی می‌کنند} \\ a = 2 \rightarrow f(x) = \sqrt{2(x)-1} = 3 \checkmark \end{cases}$ (2)

$fy - 2m = n \rightarrow a = \frac{m}{f} \quad f'(1) = \frac{m}{f} \quad f(x) = \frac{x^2 + 4x + 2m - 1}{(x+3)^2} \rightarrow f'(1) = \frac{4+2m}{19} \quad \frac{m+4}{19} = \frac{m}{f}$
 $m=2 \quad f(x) = \frac{x^2 + 2x + 1}{x+3} \rightarrow f(1) = 1 \quad f(1) - 3f(1) = n \quad n=1 \quad m+n = 2+1 = 3$ (3)

$g(x) - f(x) = \frac{9}{3 + \sin x} - \frac{2V - \sin^2 x}{9 - \sin^2 x} = \frac{2V - 9 \sin x - 2V + \sin^2 x}{9 - \sin^2 x} = \frac{\sin^2 x - 9 \sin x}{9 - \sin^2 x} = \frac{\sin x (\sin^2 x - 9)}{9 - \sin^2 x}$
 $= -\sin x \quad g'(x) - f'(x) = -\cos x \quad g'(\frac{2\pi}{3}) - f'(\frac{2\pi}{3}) = -\cos \frac{2\pi}{3} = -\frac{1}{2}$ (4)

$x > 0 \rightarrow fog(x) = f(\frac{1}{fx}) = -\frac{1}{\sqrt{\frac{1}{3x^2} + |\frac{1}{3x^2}|}} = -\frac{1}{\sqrt{\frac{2}{3x^2}}} = -\frac{1}{\frac{2}{\sqrt{3}x}} = -\frac{\sqrt{3}x}{2}$
 $(fog)'(x) = -1 \rightarrow (fog)'(\frac{2\sqrt{3}}{3}) = -1$ (5)

$g(x) = \frac{f(x)-1}{x} \quad \lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} \frac{f(x)-1}{x} \quad \frac{f(0)-1}{0} \rightarrow f'(0) \quad f(0) = 2 \times (-1) \times 2 = -4$
 $f'(x) = 2 \frac{(-1 + \sin x)}{1 + \sin x} \left(\frac{\cos x (1 + \sin x) - (-1 + \sin x) \cos x}{(1 + \sin x)^2} \right) = 2 \frac{(-1 + \sin x)}{1 + \sin x} \left(\frac{2 \cos x}{(1 + \sin x)^2} \right)$ (6)

$y = x^2 + 1 \xrightarrow{\text{قرینه نسبت به محور x}} y = -(x^2 + 1) \quad y' = -2x$
 خط d منحنی را در 2xهای قرینه قطع می‌کند پس شیب در آن نقاط 2a
 $2a(-2a) = -1 \quad -4a^2 = -1 \quad a = \pm \frac{1}{2} \quad f(\frac{1}{2}) = f(-\frac{1}{2}) = -(\frac{1}{4} + 1) = -\frac{5}{4}$
 $d: y = -\frac{a}{f} \rightarrow \text{خط د از مبدأ مختصات} = \frac{a}{f}$ (7)

$f(x) = 8x^{\frac{2}{3}} + 9x^{\frac{1}{3}} \quad f'(x) = \frac{16}{3}x^{-\frac{1}{3}} + 3x^{-\frac{2}{3}} \quad d: y = ax \quad f'(x) = a \quad 8x^{\frac{2}{3}} + 9x^{\frac{1}{3}} = \frac{16}{3}x^{-\frac{1}{3}} + 3x^{-\frac{2}{3}}(x)$
 $8x^{\frac{2}{3}} + 9x^{\frac{1}{3}} = \frac{16}{3}x^{-\frac{1}{3}} + 3x^{\frac{1}{3}} \quad \frac{16}{3}x^{-\frac{1}{3}} - 8x^{\frac{2}{3}} = 9x^{\frac{1}{3}} - 3x^{\frac{1}{3}} \quad 8x^{\frac{2}{3}}(x-2) = 3x^{\frac{1}{3}}(2-1) \quad 12x^{\frac{2}{3}} = 3x^{\frac{1}{3}}$
 $4x^{\frac{2}{3}} = x^{\frac{1}{3}} \rightarrow \begin{cases} x=0 \\ 4x^{\frac{2}{3}} = 1 \rightarrow x = \pm \frac{1}{4} \end{cases} \quad D_f = [0, +\infty) \rightarrow x = \frac{1}{4} \quad f'(x) = \frac{16}{3}\sqrt[3]{x} + \frac{3}{\sqrt[3]{x}} \rightarrow f'(\frac{1}{4}) = 8\sqrt[3]{2} + 12\sqrt[3]{2} = 20\sqrt[3]{2}$ (8)

$f(x) = ax = \frac{\sqrt{x}}{-2x^2 + x + 1}$
 $a\sqrt{x} = \frac{1}{-2x^2 + x + 1} \quad -2ax^{\frac{3}{2}} + ax^{\frac{1}{2}} + ax^{\frac{1}{2}} = 1 \quad -2ax^{\frac{3}{2}} + ax^{\frac{1}{2}} + ax^{\frac{1}{2}} - 1 = 0 \xrightarrow{\text{مشتق}} -2ax^{\frac{1}{2}} + \frac{1}{2}ax^{\frac{1}{2}} + \frac{1}{2}ax^{\frac{1}{2}} = 0$
 $-2ax^{\frac{1}{2}} + \frac{1}{2}ax^{\frac{1}{2}} + \frac{1}{2}ax^{\frac{1}{2}} = 0 \rightarrow -10ax^{\frac{1}{2}} + \frac{1}{2}ax^{\frac{1}{2}} + a = 0 \quad \Delta = 0 \rightarrow \begin{cases} x = -\frac{1}{2} \text{ در دامنه نیست} \\ x = \frac{1}{4} \rightarrow f(\frac{1}{4}) = \frac{\sqrt{\frac{1}{4}}}{1} = \frac{1}{2} \end{cases}$ (9)

$(fog)'(\frac{\sqrt{a}}{p}) = g'(\frac{\sqrt{a}}{p}) \cdot f'(g(\frac{\sqrt{a}}{p})) \quad g(x) = (x^2 - 1)^{\frac{1}{2}} \rightarrow g'(x) = \frac{1}{2}(x^2 - 1)^{-\frac{1}{2}}(2x) \quad g'(\frac{\sqrt{a}}{p}) = -\sqrt{a}$
 $g(\frac{\sqrt{a}}{p}) = 2^+ \quad f(2^+) = (2x)^+ = 4x^+ \quad f'(2^+) = 2f'x^+ = 4x^+ \quad \frac{4x^+(-\sqrt{a})}{-4\sqrt{a}x^+} = 1$ (10)