تمرین‌ها

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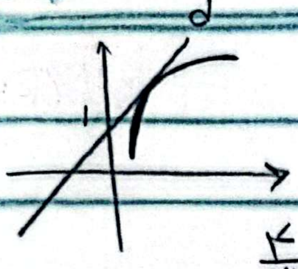

(1)

$(3,5) \to d \Rightarrow y = ax+1 \xrightarrow{(3,5)} 3a+1=5 \to$

$3a=4 \to a=\frac{4}{3} \to \frac{4}{3}x+1$

$\frac{4}{3}x+1 \simeq f(x) \to$ در نقطه $3 \to f'(3)=\frac{4}{3}$

(2)

$f(x)=\sqrt{ax-1}$

$\left.\begin{array}{l} 2a+b=2 \\ -a+b=1 \end{array}\right\} \to 3a=1 \to a=\frac{1}{3} \quad b=\frac{4}{3} \to \frac{1}{3}x+\frac{4}{3}=y$ خط مماس

$\left(\frac{1}{3}x+\frac{4}{3}\right)^2 = ax-1 \longrightarrow \frac{1}{9}x^2+\frac{16}{9}+\frac{8}{9}x = ax-1 \Rightarrow$

$x^2+16-8x = 9ax-9 \Rightarrow x^2+(8-9a)x+25=0$

$\Delta=0 \to (8-9a)^2-100=0 \Rightarrow 8-9a=\pm 10 \Rightarrow a=2$

$f(x)=\sqrt{2x-1} \to \sqrt{2\times 5-1} = 3$

(3)

$y=\frac{x^2+mx+1}{x+3} \to \frac{(2x+m)(x+3)-(x^2+mx+1)}{(x+3)^2} =$

$\frac{2x^2+6x+\cancel{mx}+3m-x^2-\cancel{mx}-1}{(x+3)^2} = \frac{x^2+6x+3m-1}{(x+3)^2}$

$4y-3x=n \to y=\frac{3}{4}x+\frac{n}{4}$

$\left.\begin{array}{l} \frac{3}{4}=\frac{1+6+3m-1}{16} \Rightarrow 12=6+3m \to m=2 \\ f(1)=\frac{1+2+1}{1+3}=1 \Rightarrow 1=\frac{3}{4}+\frac{n}{4} \to n=1 \end{array}\right\} m+n=3$

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4)

$$f(x)=\frac{27-\sin^3 x}{9-\sin^2 x}=\frac{(3-\cancel{\sin x})(9+\sin^2 x+3\sin x)}{(3-\sin x)(3+\sin x)}=\frac{9+\sin^2 x+3\sin x}{3+\sin x}$$

$$g_{(x)}=\frac{3}{\sin x+3}$$

$$h(x)=3g(x)-f(x)\longrightarrow\frac{9}{\sin x+3}-\frac{9+\sin^2 x+3\sin x}{3+\sin x}=\frac{-\sin(\sin x+3)}{\sin x+3}$$

$$=-\sin x=h(x)$$

$$h'(x)=-\cos x\to h'\left(\frac{5\pi}{3}\right)=-\cos\frac{5\pi}{3}=\frac{-1}{2}$$

5)

$$f(x)=\frac{-1}{\sqrt[5]{x+|x|}}\qquad g(x)=\frac{1}{x^5+|x^5|}$$

$$(f\circ g)_{(x)}=\frac{-1}{\sqrt[5]{\frac{2}{x^5+|x^5|}}}\Rightarrow\frac{-1}{\sqrt[5]{\frac{2}{2x^5}}}=\frac{-1}{\sqrt[5]{\frac{1}{x^5}}}\Rightarrow -x$$

$$(f\circ g)'=-1$$

6)

$$f(x)=\left(\frac{-1+\sin x}{1+\sin x}\right)^2\qquad f(x)=xg(x)+1$$

$$f'(x)=2\left(\frac{(\cos x)(1+\sin x)-(\cos x)(-1+\sin x)}{(1+\sin)^2}\right)\left(\frac{-1+\sin x}{1+\sin x}\right)$$

$$f'(x)=2\times\frac{2\cos x}{(1+\sin)^2}\times\frac{-1+\sin x}{1+\sin x}\ \star$$

$$f'(x)=\cancel{xg'(x)}^{0}+g(x)\xrightarrow[\star]{x=0}2\times\frac{2}{1}\times\frac{-1}{1}=g(x)\Rightarrow g(x)=-4$$

$$\lim_{x\to 0}g(x)=-4$$

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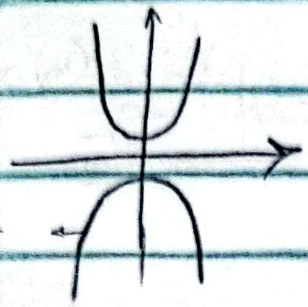

$f(x) = -x^2-1$

۷) چون x^2 است خط مماس بر آن در دو طرف قرینه می‌شود و در صورت سوال گفته شده که عمود اند شرط عمود بودن این است که شیب‌ها قرینه و معکوس هم باشند پس اعدادی که از این شرط پیروی می‌کنند ۱ و ۱- است

$f'(x) = -2x$

$-2x = 1 \rightarrow x = -\frac{1}{2}, \quad -2x = -1 \rightarrow x = \frac{1}{2}$

$x = \pm\frac{1}{2} \rightarrow f(x) = -x^2-1 \rightarrow -\frac{1}{4} - 1 \rightarrow -\frac{5}{4}$

$\frac{5}{4}$ فاصله [illegible]

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۸) $f(x) = 2\sqrt{x}(4x^2+3) \rightarrow 8x^{\frac{5}{2}} + 6x^{\frac{1}{2}} \rightarrow f'(x) = 20x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}$

$\lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = 20x^{\frac{3}{2}} + 3x^{-\frac{1}{2}} \rightarrow 20x^{\frac{5}{2}} + 3x^{\frac{1}{2}} = 8x^{\frac{5}{2}} + 6x^{\frac{1}{2}} \rightarrow$

$3x^{\frac{1}{2}} = 12x^{\frac{5}{2}} \rightarrow 1 = 4x^2 \rightarrow x = \frac{1}{2}$

$f'\left(\frac{1}{2}\right) = 20 \times \frac{1}{2\sqrt{2}} + 3\sqrt{2} = 5\sqrt{2} + 3\sqrt{2} = 8\sqrt{2}$

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۹) $f(x) = \frac{\sqrt{x}}{-2x^2+x+1} \rightarrow \frac{\frac{1}{2\sqrt{x}}(-2x^2+x+1) - (-4x+1)\sqrt{x}}{(-2x^2+x+1)^2}$

$\lim_{a\to 0} \frac{f(a)-f(0)}{a-0} = f'(a) \rightarrow af'(a) = f(a)$

$$\frac{\sqrt{a}}{-2a^2+a+1} = \frac{a\left(\frac{1}{2\sqrt{a}}(-2a^2+a+1) - (-4a+1)\sqrt{a}\right)}{(-2a^2+a+1)^2}$$

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$$\sqrt{a} = \frac{\frac{\sqrt{a}}{2}(-2a^2+a+1)-(-4a+1)a\sqrt{a}}{-2a^2+a+1} \Rightarrow$$

$$\cancel{\sqrt{a}} = \frac{\cancel{\sqrt{a}}\left(\frac{1}{2}(-2a^2+a+1)-(-4a+1)a\right)}{-2a^2+a+1} \Rightarrow$$

$$-\cancel{2a^2}+a+1 = -\cancel{a^2}+\frac{a}{2}+\frac{1}{2}+\cancel{4a^2}-a \Rightarrow$$

$$5a^2-\frac{3a}{2}-\frac{1}{2} \rightarrow 10a^2-3a-1=0 \longrightarrow \frac{+3\pm\sqrt{9+40}}{20}$$

$\sqrt{9+40} = 7$

$\frac{1}{2}$ ✓

$\frac{-4}{20}$ مردود منفی است

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b

$$\left.\begin{array}{l} f(x) = (x[x])^3 \\ g(x) = \dfrac{1}{\sqrt{x^2-1}} \end{array}\right\} \to fog \to \left(\frac{1}{\sqrt{x^2-1}}\left[\frac{1}{\sqrt{x^2-1}}\right]\right)^3 \xrightarrow[x=\frac{\sqrt{5}}{2}]{}$$

$$\left(\frac{1}{\sqrt{x^2-1}}\left[\frac{1}{\sqrt{\frac{5}{4}-1}}\right]\right)^3 \to \left(\frac{2}{\sqrt{x^2-1}}\right)^3 = fog$$

$$fog' = 3\left(\frac{2}{\sqrt{x^2-1}}\right)^2\left(\frac{-1}{2}\right)(2x)(x^2-1)^{\frac{-3}{2}} \xrightarrow{x=\frac{\sqrt{5}}{2}}$$

$$3 \times \cancel{16}\,4 \times \frac{-1}{2} \times \sqrt{5} \times 8 = -192\sqrt{5}$$

$$-48\sqrt{5} = 4 \text{ برابر}$$