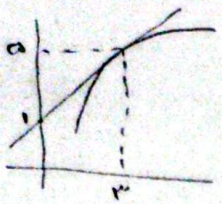
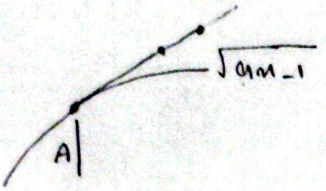
۱)



$m = \frac{a-1}{۳-۰} = \frac{۴}{۱۳}$ $\quad f'(۳) = \frac{۳}{۴}$

۲)

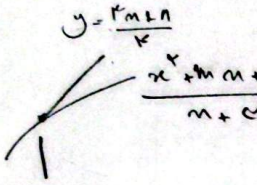


$\begin{vmatrix} ۲ & -۱ \\ ۲ & ۱ \end{vmatrix}$ $\quad$ ضمناً $m = \frac{۲-۱}{۲+۱} = \frac{۱}{۳}$ $\quad \frac{m+۴}{۳} = \sqrt{۹m-۱}$

$f(a) = \sqrt{۹a-۱}$ $\quad \frac{a}{۲\sqrt{۹m-۱}} = \frac{۱}{۳}$ $\quad \frac{a}{۲\left(\frac{m+۴}{۳}\right)} = \frac{۱}{۳}$ $\quad \frac{۳a}{۲m+۸} = \frac{۱}{۳}$ $\quad a = \frac{۲m+۸}{۹}$

$\sqrt{۹\left(\frac{۲m+۸}{۹}\right)-۱} = \sqrt{۹} = ۳$

۳)



$\frac{x^۲+mx+۱}{x+۳} \rightsquigarrow \frac{(۲x+m)(x+۳)-(x^۲+mx+۱)}{(x+۳)^۲} \rightsquigarrow \frac{۸+۴m-۲-m}{۱۶} = \frac{۳}{۴}$

$f(۱) = \frac{x^۲+۲x+۱}{x+۱} = ۲$ $\quad \frac{۲+n}{۴} = ۲$ $\quad n = ۶$ $\quad \frac{۲m+۶}{۱۶} = \frac{۳}{۴}$ $\quad m = ۲$

$m+n = ۶+۲ = ۸$

۴) $f(x) = \frac{۲\sin^۲x}{۹-\sin^۲x} = \frac{(۲\sin x)(۱+\sin^۲x+۲\sin x)}{(۳+\sin x)(۳-\sin x)}$ $\quad f'(x) = \frac{۲\sin x(-\cos x)+۲}{\cos x} = \frac{۲\times -\frac{\sqrt{۳}}{۲}\times\frac{۱}{۲}+۲}{\frac{۱}{۲}}$

$\sin\frac{۲\pi}{۳} = -\frac{\sqrt{۳}}{۲}$

$\cos\frac{۵\pi}{۳} = \frac{۱}{۲}$

$g(x) = \frac{۳}{۳+\sin x}$ $\quad g'(x) = \frac{-\cos(۰)}{(۳+\sin x)^۲} = \frac{-۳\times\frac{۱}{۲}}{(۳-\frac{\sqrt{۳}}{۲})^۲}$

۵) $(fog)' = ?$ $\quad fog = -\frac{۱}{\sqrt{\frac{۲}{x^۵}}} = -\frac{۱}{\sqrt{\frac{۱}{x^۵}}} = -\frac{۱}{\frac{۱}{x^۲\sqrt{x}}} = -x^۲\sqrt{x}$

$x > ۰$ $\quad f(x) = -\frac{۱}{\sqrt{۲x}}$ $\quad g(x) = \frac{۱}{x^۵}$

۶) $f(x) = xg(x)+۱ \rightsquigarrow f(x) = \left(\frac{-۱+\sin x}{۱+\sin x}\right)^۲ - xg(x)+۱$

$\lim_{x\to۰} g(x) = \lim_{x\to۰} \frac{\left(\frac{-۱+\sin x}{۱+\sin x}\right)^۲ - ۱}{x}$ $\quad$ هوپ $\rightsquigarrow ۲\left(\frac{-۱+\sin x}{۱+\sin x}\right)\left(\frac{\cos(۱+\sin x)-(\cos)(-۱+\sin x)}{(۱+\sin x)^۲}\right)$

$-۲\times۲ = -۴$

۷) $y = x^۲+۱$ $\xrightarrow{\text{قرینه}}$ $-x^۲-۱$ $\quad f'(a).f'(-a)$

$-\frac{۱}{۴} - ۱ = -\frac{۵}{۴}$ $\quad -۲a\times۲a = -۴a^۲$ $\quad a = \pm\frac{۱}{۲}$