تکلیف ۲۴ سهوار زارعی

۱۹٫۷۵ آفرین

NOTE: DATE: / / SUBJECT:

۱- $f'(3)$ شیب خط مماس ، شیب خط مماس $= \frac{5-1}{3-0} = \frac{4}{3}$

$f'(3) = \frac{4}{3}$ ✓ ۵

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۲- ~~شیب خط مماس~~ $\cancel{= \frac{2-1}{2+1} = \frac{1}{3}}$ $\cancel{\frac{1}{3} = \frac{a}{2\sqrt{ax_0-1}}}$ $\cancel{2\sqrt{ax_0-1} = 3a}$

$\cancel{4(ax_0-1) = 9a^2}$ $\cancel{4ax_0 - 4 = 9a^2}$ $\cancel{\frac{9a^2+4}{4a} = x_0}$

$x_0 \xrightarrow{\text{خط و تابع باهم برابرند}}$ $-\frac{1}{3} + b = 1$ $b = \frac{4}{3}$ $y = \frac{1}{3}x + \frac{4}{3}$

$\frac{1}{3}x + \frac{4}{3} = \sqrt{ax-1}$ $x+4 = 3\sqrt{ax-1}$ $(x+4)^2 = 9(ax-1)$

$x^2+16+8x = 9ax-9$ $x^2+(8-9a)x+25=0$

چون $\Delta = 0$ $(8-9a)^2 - 4(25) = 64 + 81a^2 - 144a - 100 = 0$ ۵

$81a^2 - 144a - 36 = 0$ $\div 9$ $9a^2 - 16a - 4 = 0$ $(9a+2)(a-2)$

$a = \frac{-2}{9}$ $a = 2$ قق $f(5) = \sqrt{10-1} = 3$

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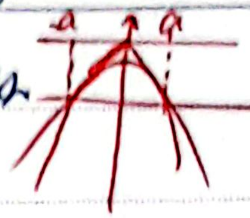
$f'(x)\ \frac{(2x-m)(x+3) - (x^2-mx+1)}{(x+3)^2} = \frac{2x^2+6x-\cancel{mx}-3m-x^2+\cancel{mx}-1}{(x+3)^2}$ ۴٫۷۵

$f'(x)\ \frac{x^2+6x-3m-1}{(x+3)^2}$ معادله شیب خط مماس $= y = \frac{3}{2}x + \frac{n}{2}$ $f'(x_0) = \frac{3}{2}$

$f'(1) = \frac{6-3m}{16} = \frac{3}{2} \Rightarrow m = -2$ $f(1) = \frac{1+2+1}{1+3} = \frac{4}{4} = 1$

$f(1) = 1$ $2(1) - 3(1) = n = 4-3 = 1$ $n = 1$ $m+n = -2+1 = -1$

$y = x^2+1 \xrightarrow[\text{نسبت به محور x}]{\text{قرینه}} -x^2-1 \Rightarrow f'(x) = -2x$ — ۷



$f(a)\, f'(-a) = -1 \Rightarrow (-2a)(2a) = -1 \overset{a>0}{\Rightarrow} a = \frac{1}{2}$

$f(\frac{1}{2}) = -((\frac{1}{2})^2+1) = -\frac{5}{4} \to g = -\frac{5}{4} \Leftarrow$ عرض از مبدأ خط مماس

فاصله از مبدأ مختصات $\frac{5}{4}$ است

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$f(x) = g(x) \to$ مماس $\Rightarrow 2\sqrt{x}(4x^2+3) = mx \Rightarrow 8x^2+6 = m\sqrt{x}$ — ۸

$f'(x) = g'(x)\quad 16x = \frac{m}{2\sqrt{x}} \Rightarrow 8x^2+6 = 32x^2 \overset{x>0}{\Rightarrow} x = \frac{1}{2} \xrightarrow[x=\frac{1}{2}]{\text{جایگذاری}}$

$m = 32x\sqrt{x} \quad m = 8\sqrt{2}$

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$f'(x) = \dfrac{\frac{1}{2\sqrt{x}}(-2x^2+x+1) - (-4x+1)\sqrt{x}}{(-2x^2+x+1)^2} \Rightarrow \dfrac{\sqrt{x}}{\frac{-2x^2+x+1}{x}} = \dfrac{\frac{-2x^2+x+1}{2\sqrt{x}} - (-4x+1)\sqrt{x}}{(-2x^2+x+1)^2}$ — ۹

$\dfrac{f(x)}{x} = f'(x) \qquad \dfrac{\sqrt{x}}{x} = \dfrac{\frac{-2x^2+x+1 - 2x(-4x+1)}{2\sqrt{x}}}{-2x^2+x+1} \Rightarrow 2 = \dfrac{9x^2-x+1}{-2x^2+x+1} \Rightarrow$

$10x^2-3x-1=0 \qquad f(\frac{1}{2}) = \dfrac{\frac{\sqrt{2}}{2}}{-2(\frac{1}{4})+\frac{1}{2}+1} = \dfrac{\sqrt{2}}{2}$

$x = -\frac{1}{5}$ ، $\frac{1}{2}$ ق ق

ق.غ.ق ← در دامنه موجود نیست

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$x \to (\frac{\sqrt{5}}{2})' \Rightarrow \sqrt{x^2-1} \to (\frac{1}{2}) \Rightarrow g(x) \to 2^+$ — ۱۰

$(f(g(x)))' = g'(\frac{\sqrt{5}}{2}) \cdot f'_+(2)$

$g'(x) = ((x^2-1)^{-\frac{1}{2}})' = -\frac{1}{2}(x^2-1)^{-\frac{3}{2}} 2x \Rightarrow g'(\frac{\sqrt{5}}{2}) = -4\sqrt{5}$

$x \to 2^+ \Rightarrow f(x) = (8x^3) \Rightarrow f'(x) = 24x^2 \Rightarrow f'_+(2) = 96 \cdot (-4\sqrt{5}) =$

$= 8(-48\sqrt{5})$ ۲ ۸ برابر

۲- الف)

$$f(x) = \frac{(3-\sin x)(9+3\sin x+\sin^2 x)}{(3-\sin x)(3+\sin x)} = \frac{9+3\sin x+\sin^2 x}{3+\sin x}$$

$$3g(x) - f(x) = 3\left(\frac{3}{3+\sin x}\right) - \frac{9+3\sin x+\sin^2 x}{3+\sin x} = \frac{-3\sin x-\sin^2 x}{3+\sin x} = -\sin x$$

$$h' = -\cos x = -\cos\left(\frac{5\pi}{3}\right) = -\frac{1}{2}$$

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$$|x|\;|x^5| = x, x^5 \Rightarrow f(x) = \frac{-1}{\sqrt[5]{2x}}, \quad g(x) = \frac{1}{2x^5}$$

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$$f(g(x)) = \frac{-1}{\sqrt[5]{2\times\frac{1}{2x^5}}} \Rightarrow f(g(x)) = -x \xrightarrow{\text{مشتق}} (f(g(x)))' = -1$$

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$$x(g(x)) = f(x) - 1 \Rightarrow g(x) = \frac{f(x)-1}{x} \Rightarrow g(x) = \frac{\left(\frac{-1+\sin x}{1+\sin x}\right)^2 - 1}{x}$$

$$f(x) - 1 = \left(\frac{-1+\sin x}{1+\sin x} - 1\right)\left(\frac{-1+\sin x}{1+\sin x} + 1\right) \Rightarrow \left(\frac{-2}{1+\sin x}\right)\left(\frac{2\sin x}{1+\sin x}\right) = \frac{-4\sin x}{(1+\sin x)^2}$$

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$$\lim_{x\to 0} g(x) = \frac{\frac{-4\sin x}{(1+\sin x)^2}}{x} = \lim_{x\to 0}\left(\frac{-4}{(1+\sin x)^2} \times \frac{\sin x}{x}\right) = -4 \quad \lim_{x\to 0} g(x) = -4$$

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$x=1 \longrightarrow y=\frac{۲+m}{۴}$

$y'=\frac{(\overset{m+۲}{\cancel{۲x+m}})(\overset{۴}{\cancel{x+۳}})-(\overset{m+۲}{\cancel{x^{۲}+mx}}+1)}{(\underset{۴}{\cancel{x+۳}})^{۲}}=\frac{۳m+۶}{۱۶}=\frac{۳}{۴} \rightsquigarrow \boxed{m=۲}$

$y=\frac{۳}{۴}x+\frac{n}{۴} \rightsquigarrow \frac{۳+n}{۴}=\frac{۲+۲}{۴} \rightsquigarrow \boxed{n=1}$

$\boxed{m+n=۳}$