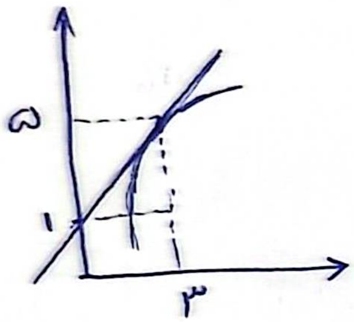
باران تسلیمی

۱ـ



$f'(۲) =$ شیب خط $= \frac{\Delta y}{\Delta x}$

$\Rightarrow \frac{۴}{۳}$

۲-

$\text{شیب} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ $\quad \lim_{x \to ۵} \frac{f(x) - f(۵)}{x - ۵} = \frac{۱}{۳}$

$y = \frac{x + ۴}{۳}$ $\quad f(x) = \sqrt{ax - ۱}$ $\quad f'(۵) = \frac{۱}{۳} \to \frac{a}{۲\sqrt{۵a - ۱}} = \frac{۱}{۳}$

$۳a = ۲\sqrt{۵a - ۱} \longrightarrow ۹a^۲ = ۲۰a - ۴ \longrightarrow a = ۲$

$f(۵) = \sqrt{۲(۵) - ۱} = \boxed{۳}$

$$y' = \frac{3}{4} \rightarrow \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2} = \frac{3}{4} \xrightarrow{x=1}$$

$$\frac{4(m+2)-(m+2)}{16} = \frac{3}{4} \longrightarrow m+2 = 4 \longrightarrow \boxed{m=2}$$

$$y(1) = \frac{m+2}{4} = 1 \qquad 4-3 = n \longrightarrow \boxed{n=1}$$

$$\boxed{m+n=3}$$

-۴

$$f(x) = \frac{\sin^{۲} x + ۳\sin x + ۹}{۳ + \sin x}$$

$$g(x) = \frac{۳}{۳ + \sin x}$$

$$(۳g - f)' = ?$$

$$\frac{\cancel{۹} - \sin^{۲} x - ۳\sin x - \cancel{۹}}{۳ + \sin x} = \frac{-\sin x\,(\sin x + ۳)}{\sin x + ۳} = -\sin x \xrightarrow{\text{مشتق}} -\cos x$$

$$-\cos\left(\frac{۵\pi}{۳}\right) = \frac{-۱}{۲}$$

$$f(x) = -\frac{1}{\sqrt[۵]{x+|x|}} \qquad g(x) = \frac{1}{x^{۵}+|x^{۵}|}$$ ۵-

؟ $g'(\sqrt[۵]{۳})\, f'(g(\sqrt[۵]{۳}))$ مقدار

$= (f\circ g)'(x)$

$$x = \sqrt[۵]{۳} \to g(x) = \frac{1}{۲x^{۵}}$$

$\Rightarrow$

$$f(x) = \frac{-1}{\sqrt[۵]{۲x}} \longrightarrow f\circ g(x) = -x \to (f\circ g)'(\sqrt[۵]{۳}) = -1$$

۶-

$$\lim_{x\to 0} g(x) = ?$$

$$f(x) = x\,g(x) + 1 \quad , \quad f(x) = \left(\frac{-1+\sin x}{1+\sin x}\right)^2$$

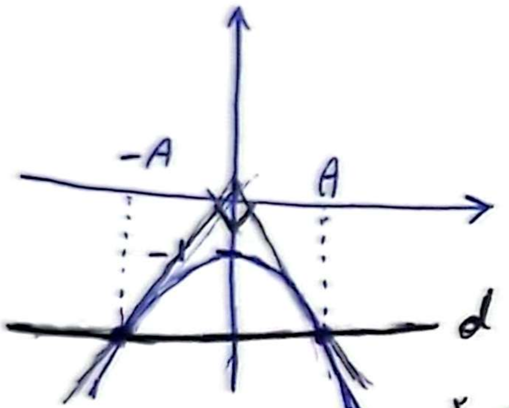
$$\hookrightarrow 1 + \frac{-2}{1+\sin x}$$

$$x\,g(x) = f(x) - 1$$

$$\lim_{x\to 0} g(x) = \frac{f(x)-1}{x} \xrightarrow{HOP} 2\left(1+\frac{-2}{1+\sin x}\right)\left(\frac{2\cos x}{(1+\sin x)^2}\right) \quad \frac{2}{1}$$

$$\xrightarrow{x\to 0} 2(-1)(2) = -4$$

۷-



$y = -x^2 - 1$

$f'(x) = -2x$

$f'(A) = \frac{-1}{f'(-A)}$

$-2A = \frac{-1}{2A} \rightarrow -4A^2 = -1$

$A = \frac{1}{2}$

$y = -\left(\frac{1}{2}\right)^2 - 1 \rightarrow \frac{-1}{4} - \frac{4}{4} = \frac{-5}{4}$

فاصله خط d تا محور xها. پس $\left|\frac{-5}{4}\right| = 1{,}25$

۸-

شیب $= \dfrac{\Delta y}{\Delta x} \longrightarrow \dfrac{2\sqrt{x}\,(4x^2+3)}{x}$

$\begin{vmatrix} 0 \\ 0 \end{vmatrix} \quad \begin{vmatrix} x \\ 2\sqrt{x}\,(4x^2+3) \end{vmatrix}$

$\longrightarrow \underbrace{\dfrac{20x^2+3}{\sqrt{x}}}_{\text{مشتق تابع } f} = \dfrac{2\sqrt{x}\,(4x^2+3)}{x} \rightsquigarrow 20x^3+3x = 2x(4x^2+3)$

$20x^3+3x = 8x^3+6x$

$12x^3-3x = 0$

$3x(4x^2-1) = 0$

$3x(2x+1)(2x-1) = 0$

$x=0$ ✗ $\quad x=\frac{-1}{2}$ ✗ $\quad x=\frac{1}{2}$ ✓

$\dfrac{\sqrt{2}\,(4)}{\frac{1}{2}} = 8\sqrt{2} \Rightarrow$ شیب

-۹

$d \Rightarrow y = ax$

$\sqrt{x} = t \qquad f(x) = \dfrac{t}{-۲t^{۴} + t^{۲} + ۱} = at^{۲}$

$-۲at^{۵} + at^{۳} + at - ۱ = ۰$

$\rightarrow -۱۰at^{۴} + ۳at + a = ۰ \rightarrow -a(۱۰t^{۴} - ۳t^{۲} - ۱) = ۰$

$\begin{cases} t^{۲} = \frac{۱}{۲} \checkmark \\ t^{۲} = \frac{-۱}{۵} \times \end{cases} \qquad f(\tfrac{۱}{۲}) = \dfrac{\frac{۱}{\sqrt{۲}}}{-۲(\frac{۱}{۴}) + \frac{۱}{۲} + ۱} = \dfrac{۱}{\sqrt{۲}}$

-۱۰

$f(x) = (x[x])^{۳} \qquad g(x) = \dfrac{۱}{\sqrt{x^{۲} - ۱}}$

$(f \circ g)'(\frac{\sqrt{۵}}{۲}) = g' \times f'(g) = \dfrac{\cancel{۲۷}\cancel{x^{۲}} \times \frac{۵}{۴} \times \cancel{۴} \times \cancel{\sqrt{۵}}}{\cancel{-۴۸} \times \cancel{\sqrt{۵}}} = \dfrac{-۵}{۱۶}$

$-۴۸\sqrt{۵}$ چند برابر -

$g' = \dfrac{-\frac{۲x}{۲\sqrt{x^{۲}-۱}}}{x^{۲}-۱} = \dfrac{-x}{\sqrt{x^{۲}-۱} \times x^{۲}-۱} = \dfrac{\frac{-\sqrt{۵}}{۲}}{\frac{۱}{۸}} = -۴\sqrt{۵}$

$(-۹x)^{۳} = -۲۷x^{۳} \xrightarrow{\text{مشتق}} -۸۱x^{۲} \xrightarrow{x = \frac{\sqrt{۵}}{۲}} -۸۱ \times \frac{۵}{۴}$