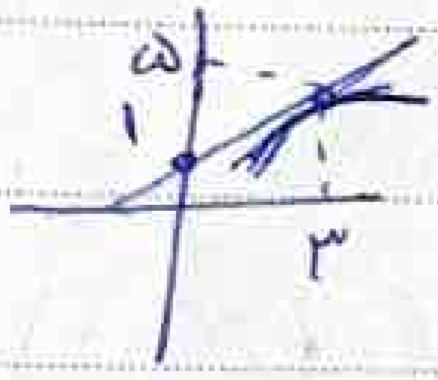
ثنا امیرحاتمی    تکلیف ۲۴    ۱۷/۷۵




سوال ۱    ۲ نقطه از خط مماس را داریم: (۰ ، ۱) ، (۳ ، ۵)

$m=\frac{4}{3}=f'(3)$

سوال ۲

$(-1\ \ 1)\ (2\ \ 2) \rightarrow m=\frac{2-1}{2+1}=\frac{1}{3} \rightarrow d: \frac{1}{3}x+\frac{4}{3} \rightarrow f'(c)=\frac{1}{3}$

$A\left(c\ \ \frac{1}{3}c+\frac{4}{3}\right) \qquad f(c)=\sqrt{ac-1}=\frac{c}{3}+\frac{4}{3}$

$f'(c)=\frac{1}{3}=\frac{a}{2\sqrt{ac-1}} \Rightarrow \frac{1}{3}=\frac{a}{2\left(\frac{c}{3}+\frac{4}{3}\right)} \rightarrow a=\frac{2c+8}{9}$

$f(c)=\sqrt{\frac{2c^2+8c-9}{9}}=\frac{c+4}{3} \xrightarrow{\text{توان } 2} 2c^2+8c-9=c^2+8c+16 \rightarrow c=5$

$f(5)=\frac{5}{3}+\frac{4}{3}=3$    ۳

سوال ۳    ۱/۷۵

$f'(x)=\frac{(2x+m)(x+3)-(x^2+mx+1)}{(x+3)^2}=\frac{x^2+6x+3m-1}{(x+3)^2}$

$f'(1)=\frac{7+3m-1}{16}=\frac{3}{4} \rightarrow m=2 \rightarrow f(1)=\frac{1+2+1}{4}=1$

$d: y=\frac{3}{4}x+\frac{n}{4} \rightarrow \frac{3}{4}+\frac{n}{4}=1 \rightarrow n=4 \qquad m+n=6$

سوال ۴

$$3g(x) - f(x) = \frac{9}{3+\sin x} - \frac{\cancel{(3-\sin x)}(9+\sin^2 x+3\sin x)}{\cancel{(3-\sin x)}(3+\sin x)}$$

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$$= \frac{-(\sin^2 x + 3\sin x)}{3+\sin x} = -\sin x \qquad (3g-f)'\left(\frac{5\pi}{3}\right) = -\cos\left(\frac{5\pi}{3}\right) = -\frac{1}{2}$$

سوال ۵

$x = \sqrt[5]{3} \to g(x) = \frac{1}{3x^5}$ $\quad fog(x) = \frac{-1}{\sqrt[5]{\frac{1}{3x^5}+\frac{1}{3x^5}}}$

$\Rightarrow fog(x) = -\sqrt[5]{x^5} = -x \to (fog(x))' = -1 = g'(\sqrt[5]{3})\, f'(g(\sqrt[5]{3}))$

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سوال ۶

$f(x) = \frac{\sin^2 x - 2\sin x + 1}{\sin^2 x + 2\sin x + 1}$ $\quad f(x) = x\,g(x) + 1$

$\to x \times g(x) = \frac{-4\sin x}{\sin^2 x + 2\sin x + 1} \to g(x) = \frac{1}{x} \times \frac{-4\sin x}{\sin^2 x + 2\sin x + 1}$

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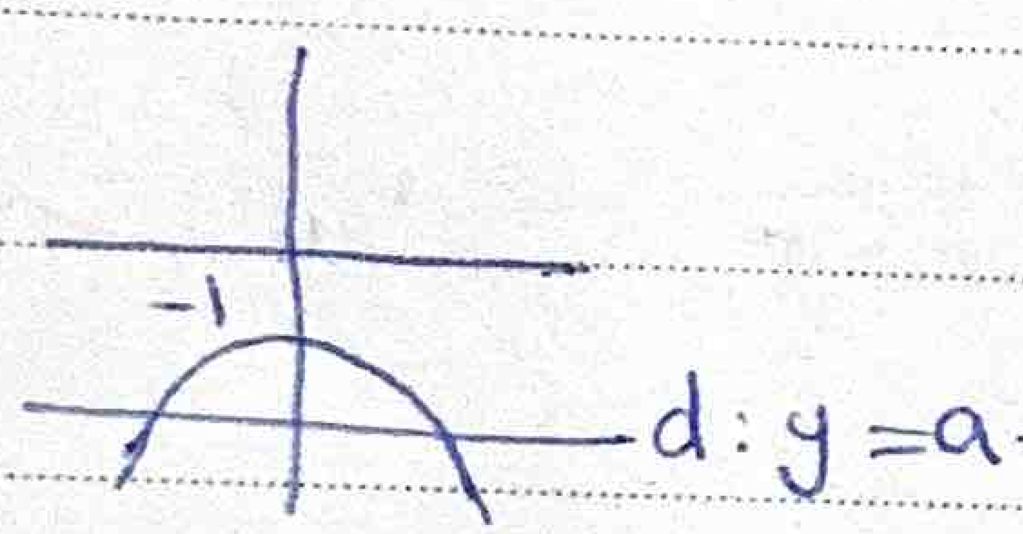
$\lim_{x \to 0} g(x) \xrightarrow{\text{هم‌ارزی}} \frac{-4x}{x^3 + 2x^2 + x} \xrightarrow{\text{Hop}} \frac{-4}{3x^2 + 4x + 1} \to \boxed{-4}$

سوال ۷




$a < -1$ $\quad -x^2 - 1 = a \to x = \pm\sqrt{-1-a}$

$f'_{(x)} = -2x \to -2(\sqrt{-1-a}) \times -2(-\sqrt{-1-a}) = -1$

$\to -1 - a = \frac{1}{4} \to a = \frac{-5}{4} \Rightarrow$ ~~فاصله~~ $d$ از مبدأ $= \frac{5}{4}$

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سوال ۸

$d : y = ax$ $\quad f(x) = 2\sqrt{x}\,(4x^2 + 3) = ax$

$f' = \frac{4x^2+3}{\sqrt{x}} + 8x(2\sqrt{x}) = \frac{20x^2+3}{\sqrt{x}} = a$

$\to 8x^2 + 6 = 20x^2 + 3$

$x^2 = \frac{3}{12} = \frac{1}{4} \xrightarrow{x>0} x = \frac{1}{2} \to a = \frac{8}{\frac{1}{\sqrt{2}}} = 8\sqrt{2}$

۲

سیرا دهقان

Date: Sub:

سوال ۱۵

$x < \frac{\sqrt{5}}{2} \to x^2 < \frac{5}{4} \to x^2-1 < \frac{1}{4} \to \frac{1}{x^2-1} > 4$

$\to \frac{1}{\sqrt{x^2-1}} > 2$ ms $fog(x) = 8 \times (x^2-1)^{-\frac{3}{2}}$

$\xrightarrow{\text{مشتق}} 8 \times \left(-\frac{3}{2}\right)(2x)(x^2-1)^{-\frac{5}{2}} \xrightarrow{x=\frac{\sqrt{5}}{2}}$ ۲

12 32

$8 \times \frac{-3}{\cancel{2}} \times \sqrt{5} \times \cancel{\left(\frac{-1}{4}\right)^{-\frac{5}{2}}} = -48\sqrt{5} \times 8$

۸ برابر $-48\sqrt{5}$ است

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$x=1 \longrightarrow y=\dfrac{۲+m}{۴}$

$y'=\dfrac{(۲\cancel{x}+m)(\cancel{x}+۳) - (x^۲+\cancel{mx}+1)}{(\cancel{x}+۳)^۲} = \dfrac{۳m+۹}{۱۶} = \dfrac{۳}{۴} \rightsquigarrow \boxed{m=۲}$

$y=\dfrac{۳}{۴}x+\dfrac{n}{۴} \rightsquigarrow \dfrac{۳+n}{۴}=\dfrac{۲+۲}{۴} \rightsquigarrow \boxed{n=1}$

$\boxed{m+n=۳}$

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$y=mx \longrightarrow \dfrac{\sqrt{a}}{-۲a^۲+a+1}=ma \longrightarrow \dfrac{1}{-۲a^۲+a+1}=m\sqrt{a}$

$m\sqrt{a}(-۲a^۲+a+1)=1 \longrightarrow -۲m(a^{\frac{۵}{۲}})+m(a^{\frac{۳}{۲}})+m(a)^{\frac{1}{۲}}=1$ مشتق $\rightsquigarrow$

$-۵m(a^{\frac{۳}{۲}})+\frac{۳}{۲}m(a^{\frac{1}{۲}})+\frac{m}{۲}(a^{-\frac{1}{۲}})=0$

$\frac{m}{۲}(a^{-\frac{1}{۲}})(-10a^۲+۳a+1)=0 \rightsquigarrow a=-\frac{1}{۵}$ یا $a=\frac{1}{۲}$ $(a>0)$

$f(a)=\dfrac{\frac{\sqrt{۲}}{۲}}{-۲(\frac{1}{۴})+\frac{1}{۲}+1}=\dfrac{\frac{\sqrt{۲}}{۲}}{1}=\boxed{\frac{\sqrt{۲}}{۲}}$