حل مسئله

① $\frac{f(2)-f(1)}{2-1} = \frac{\left(1-\frac{a}{2}\right)-(1-a)}{1} = \frac{\frac{a}{2}}{1} = \frac{a}{2}$ شیب وتر $= \frac{a}{2}$

$f'(x) = -ax - 1 \times x^{-2} = ax^{-2} \longrightarrow \frac{a}{3} = \frac{a}{x^2} \rightarrow x^2 = 3 \rightarrow x = \pm\sqrt{3}$

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② شیب خط $y = x$ برابر 1 است

$y' = 2ax - a = 1 \rightarrow 2ax = a+1 \rightarrow ax = \frac{a+1}{2} \rightarrow a(a+1) = \frac{3}{2}$

$a = \pm\frac{1}{2}$

$x = 2ax^2 - ax + \ln a$

$0 = 2ax^2 - ax + \ln a \rightarrow ax^2 - ax + a = 0 \rightarrow ax^2 - ax + a = 0$

$a = -\frac{1}{2}$

$a(x^2 - x + 1) = 0$

$x^2 - x + 1 = 0$

$\Delta = 1 - 4 = -3$

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③

$y' = 3x^2 - 12 = 0 \rightarrow x = \pm 2$

x	-2		2	
y'	$+$	0	$-$	0 $+$
y	↗		↘	↗

$x = 2 \rightarrow y = 8 - 24 + 2 = -14$

مقدار min نسبی

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④

$y' = 3x^2 + 2ax - 2b \longrightarrow x = 0 \rightarrow y' = 0 : 2b = a$

$x = -2 \rightarrow y' = 0 : 12 - 4a = 0 \rightarrow a = 3$

$y = x^3 + 3x^2 - 4$

$\begin{vmatrix} 0 & -2 \\ -4 & 0 \end{vmatrix}$

$d = \sqrt{(0-(-2))^2 + (-4-0)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

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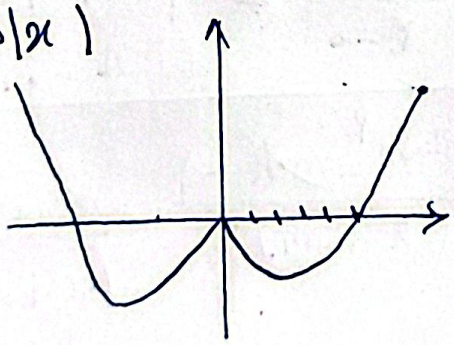
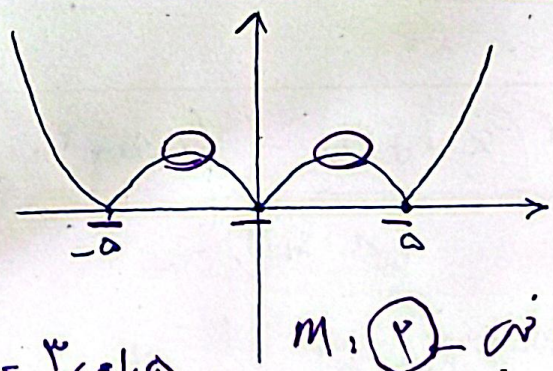
⑤

$x^2 - 5|x|$





$\frac{m}{n} = \frac{4}{3} = 1.3$

m: ② ← نسبی max

n: ③ ← نسبی min

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⑥

$f(x) \begin{cases} x \geq 0 \longrightarrow x^2 + 3x \\ x < 0 \longrightarrow -x^2 + 3x \end{cases}$

$x^2 + 3x = 0 \rightarrow x = 0, -3$

$2x + 3 = 0 \rightarrow x = -\frac{3}{2}$

$-2x + 3 = 0 \rightarrow x = \frac{3}{2}$

$|f(x)|$ نسبی $\rightarrow f(x) = 0$

$f(x) = 0 \longrightarrow x = 0, -3, \frac{3}{2}$

تنها جا دارد: $x = 0$

(۸)

$[0,a]$: $f(x) = \sqrt[3]{x^2}(-x+a) = -x^{\frac{5}{3}} + a x^{\frac{2}{3}}$

$\longrightarrow f'(x)=0 : -\frac{5}{3}x^{\frac{2}{3}} + \frac{2}{3}a x^{-\frac{1}{3}} = 0$

$\longrightarrow -\frac{1}{3}x^{-\frac{1}{3}}\left(\frac{5}{3}x + 2a\right) = 0$ ↗ $x=0$ ممکن ✓ ↘ $x = -\frac{6a}{5}$

$\left(\sqrt[3]{\frac{a^2}{2a}}\right)\left(\frac{2a}{a}\right) = \frac{1}{2}$

$\sqrt[3]{\frac{4a^3}{a^3}} = \frac{1}{2} \longrightarrow \frac{4a^3}{a^3} = \frac{1}{8} \rightarrow \frac{a^3}{a^3} = a^3 \rightarrow a = \frac{1}{2}$

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(۷)

$f(x)$ $\begin{cases} x<0 \rightarrow \sqrt{x^2-x} = \sqrt{x(x-1)} \\ ? \\ x>0 \rightarrow \sqrt{-x^2-x} = \sqrt{-x(x+1)} \end{cases}$

x | -1 | 0 | 1
$x(x-1)$ | $+$ | $-$ | $+$ ✓ ممکن
$-x(x+1)$ | $-$ | $+$ | $-$ ✓ ممکن

(۱) $\frac{x}{f(x)}$ | -2 | 0^+ | 1

$\min \rightarrow 0$ ، $\max \rightarrow 1$ ، $\lim \rightarrow 1$

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(۶)

$y' = \frac{m(x-1+m) - 1(mx+2)}{(x-1+m)^2} = \frac{mx-m^2-mx-2}{(x-1+m)^2} = \frac{-m^2-2}{(x-1+m)^2}$

$x=1 \rightarrow w=0$ ممکن

$-1 \leq x \leq 1 \xrightarrow{*} -4 \leq w-1 \leq 2 \rightarrow -4 < w-1 \leq 2$

$0 < w \leq 4$ ← مشتق‌پذیر

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(۵)

$f(x)$ $\begin{cases} x<0 & \frac{x}{1-x^2} \rightarrow f'(x) = \frac{(1-x^2)1 - x(-2x)}{(1-x^2)^2} = \frac{1+x^2}{(1-x^2)^2} \\ x>0 & \frac{x}{1+x^2} \rightarrow f'(x) = \frac{(1+x^2)1 - x(2x)}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2} \end{cases}$

$x=0$ ✓ $1-x^2 \neq 0$ ← مشتق‌پذیر