سوال ۱

$$f(x) = 1 - \frac{a}{x}$$

$$\frac{f(2)-f(1)}{2-1} \longrightarrow \frac{1-\frac{a}{2}-1+a}{1} = \frac{a}{2} = \frac{1}{2} \Rightarrow a = 1$$

شرط همگرایی (باز: (۱و۲))

$$x = \pm\sqrt{2} \longleftarrow x^2 = 2 \longleftarrow \frac{x^2}{x} = \frac{2}{x} \longrightarrow \text{شیب خط} = \text{شیب خط}$$

سوال ۲

$$y = 2ax^2 - 6x + 18a$$

$$y' = 4ax - 6 \qquad y'(3) = 0 \Rightarrow 12a - 6 = 0 \Rightarrow a = \frac{1}{2}$$

$x = 3$ محل رأس سهمی

$$2ax^2 - 6x + 18a = x \longrightarrow ax^2 - 4x + 9a = 0$$

$$\Delta = 16 - 36a^2 = 0 \longrightarrow a = \pm\frac{2}{3}$$

$a = -\frac{2}{3}$ غ ق ق $\quad a = \frac{2}{3}$ ق

سوال ۳

$$y = x^2 - 12x + 36$$

$$y' = 2x - 12 = 0 \longrightarrow x = 6 \longrightarrow y = 0 \longrightarrow \text{می‌نیمم}$$

$$x = 2 \longrightarrow y = 4 - 24 + 36 = 16$$

$$x = -2 \longrightarrow y = 4 + 24 + 36 = 64$$

$$\min \begin{bmatrix} 16 \\ 0 \\ 64 \end{bmatrix}$$

سوال ۴

$$y = x^3 + 3ax^2 - 24x + 8$$

$$y' = 3x^2 + 6ax - 24 = 0 \longrightarrow x = 2 \quad 3a - 12 = 0 \Rightarrow a = 4$$

$$x = -4 \longrightarrow y = -64 + 96 + 96 + 8 = 136$$

$$x = 2 \longrightarrow y = 8 + 48 - 48 + 8 = 16$$

$$[-4, 2] \quad [0, 3]$$

$$y = x^3 + 3x^2 - 9x + 8$$

$$x = 3 \longrightarrow y = 35$$

$$x = 0 \longrightarrow y = 8$$

$$x = -4 \longrightarrow y = -64 + 48 + 36 + 8 = 28$$

$$x = 2 \longrightarrow y = 8 + 12 - 18 + 8 = 10$$

$$\text{طول} = \int \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \longrightarrow \sqrt{(5-0)^2 + (10-24)^2} = \sqrt{25 + 196} = \sqrt{221}$$

سوال ۴

$y = x^2 - 2|x| - 3$

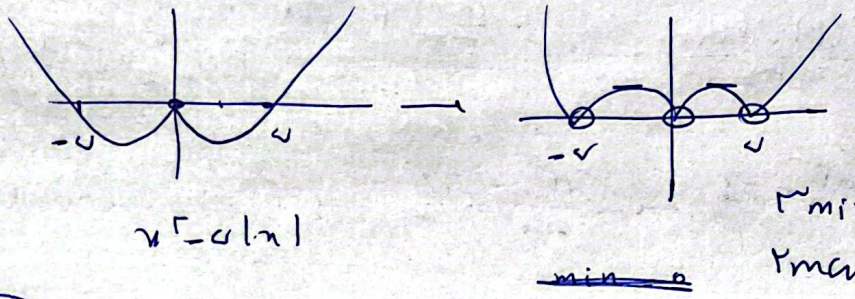
$x \geq 0$: $x^2 - 2x - 3$; $x(\text{راس}) = 1$

$x < 0$: $x^2 + 2x - 3$; $x(\text{راس}) = -1$

m max

n min

$|x^2 - 2|x| - 3|$



$|x^2 - 2|x| - 3|$

$m_{max} = 3$

$n_{min} = 0$

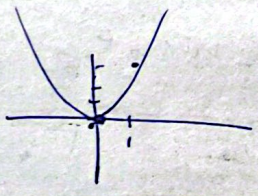
$\frac{m}{n} = \frac{3}{0}$

سوال ۵

$|x(|x| + 2)| \longrightarrow |x^2 + 2x|$

$x \geq 0$: $|x^2 + 2x - 2|$

$x < 0$: $|x^2 + 2x|$



$|x^2 + 2x|$ ، $x = 0$ ، $(-2, 0)$ ، $|x(x+2)|$

$|x(x+2)|$ ، $x = 0$

تعداد نقاط صعب $= 2$

سوال ۶

$f(x) = 2\sqrt{x^2 |x - a|}$

$|x - a| \frac{2}{3} x$

$(x+2)\frac{2}{3} x \longrightarrow \frac{2}{3} x^3 + ax^{\frac{2}{3}}$

$x = 0$ ، $x = -2$ ، $x = a$

$0 = -2 \times a^{\frac{2}{3}} + \frac{2}{3} x^{\frac{2}{3}}$

$0 = (a^{\frac{2}{3}} + x^{\frac{2}{3}}) 2 \times x^{\frac{1}{3}}$

$0 = \frac{2}{3} a + \frac{2}{3} \longrightarrow \frac{2}{3} = -\frac{2}{3} a$

$\frac{2}{3} = a \longrightarrow \frac{2}{3} = \frac{2}{3}$

سوال ۸

$y = \sqrt{x|x| - x}$

$x \to 0^+ \quad \sqrt{x^2 - x}$

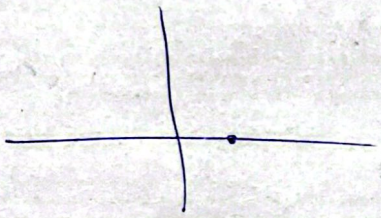
$x \to 0^- \quad \sqrt{-x^2 - x}$

$\frac{2x-1}{2\sqrt{x^2-x}}$ (+)

$x(x-1)$

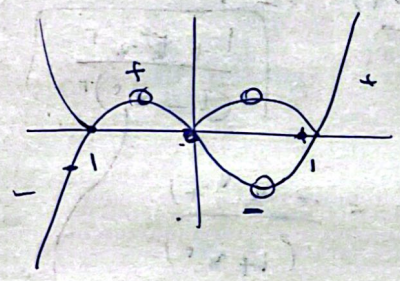
$\frac{-2x-1}{2\sqrt{-x^2-x}}$ (-)

$-x(1+x)$



$-1 \quad 0 \quad -\frac{1}{2} \quad \frac{1}{2}$

نقطه حران



$k=4$

$\min = 3$ در

$\max = 2$ در

$3 + 2\sqrt{2}$

$3 - 2\sqrt{2}$

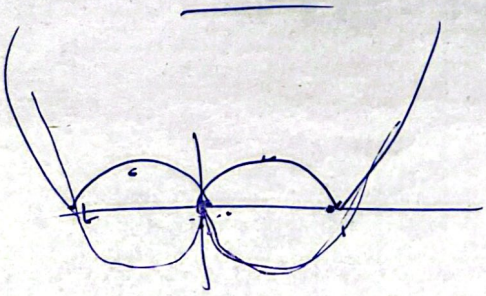
$\frac{\sqrt{2}}{2}$

$-1 \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2}$

$- \quad + \quad - \quad + \quad - \quad +$

$\searrow \quad \nearrow \quad \searrow \quad \nearrow \quad \searrow \quad \nearrow$

min max min max min



$x^2 - x$

$x(x-1)$

$x^2 - x = 0$

$x(1+x)(x-1)$

$-1 \quad 0 \quad 1$

سوال ۹

$y = \frac{mx+2}{x-1+m}$ — مشتق — $y' = \frac{m(x-1+m) - 1(mx+2)}{(x-1+m)^2}$

$y' = \frac{m^2 - m - 2}{(x-1+m)^2}$ — y' ثابت — $m^2 - m - 2 = 0$

$(m-2)(m+1) = 0$

$m = 2$ یا $m = -1$ — $x - 1 + m \neq 0 \rightarrow x \neq 1 - m$

$1 - m \neq 1 \rightarrow m \neq 0$

طبق سوال $m \neq 2$ — $2 \neq 0$ و $2 \neq 1$ — گزینه ۲

سوال ۱۵

$f(x) = \frac{|x|x-1|}{x}$

$x < 0$ — $y' = \frac{(2x-1)(x) + (x^2-x+1)}{x^2} = \frac{2x^2 - x + x^2 - x + 1}{x^2} = \frac{3x^2 - 2x + 1}{x^2}$

$x > 0$ — $y' = \frac{(2x-1)(x) - (x^2-x+1)}{x^2} = \frac{2x^2 - x - x^2 + x - 1}{x^2} = \frac{x^2 - 1}{x^2}$

$x = 1$ — $\frac{x^2-1}{x^2} = 0$

$x = -1$ — $\frac{3x^2-2x+1}{x^2} = \frac{3+2+1}{1} = 6$

$x = 0$ — $\frac{x^2-1}{x^2}$ — $\frac{0-1}{0^+} = -\infty$

$x = 0$ — $\frac{3x^2-2x+1}{x^2}$ — $\frac{1}{0^+} = +\infty$

تابع در نقطه $x = 0$ مشتق‌پذیر نیست

گزینه ۴