

۱  $\frac{f(x) - f(1)}{x^2 - 1} = f'(x) \Rightarrow \frac{(1-\frac{a}{x}) - (1-a)}{x^2 - 1} = \frac{\frac{a}{x}}{x^2 - 1} = \frac{a}{x^2} = \frac{a}{x^2} \Rightarrow \frac{1}{x^2} = \frac{1}{x^2} \Rightarrow x = \pm \sqrt{3}$

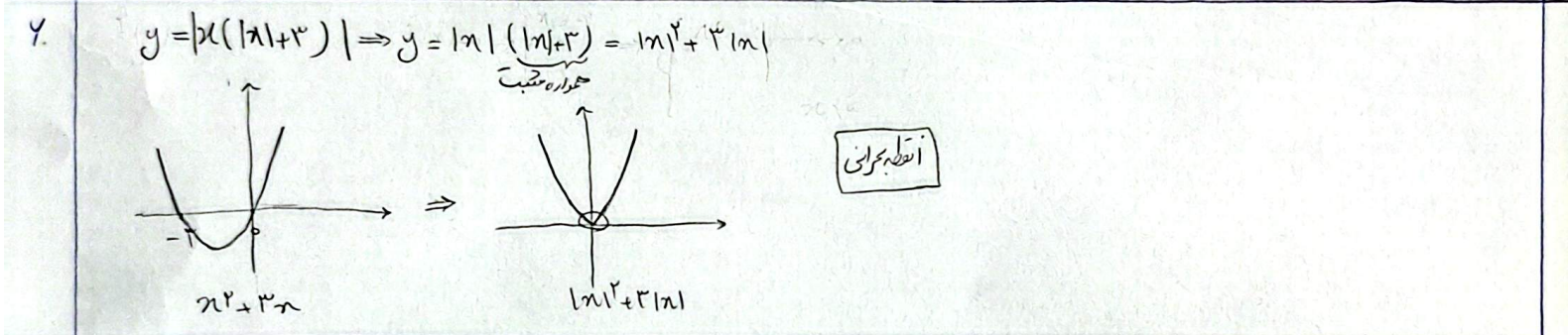
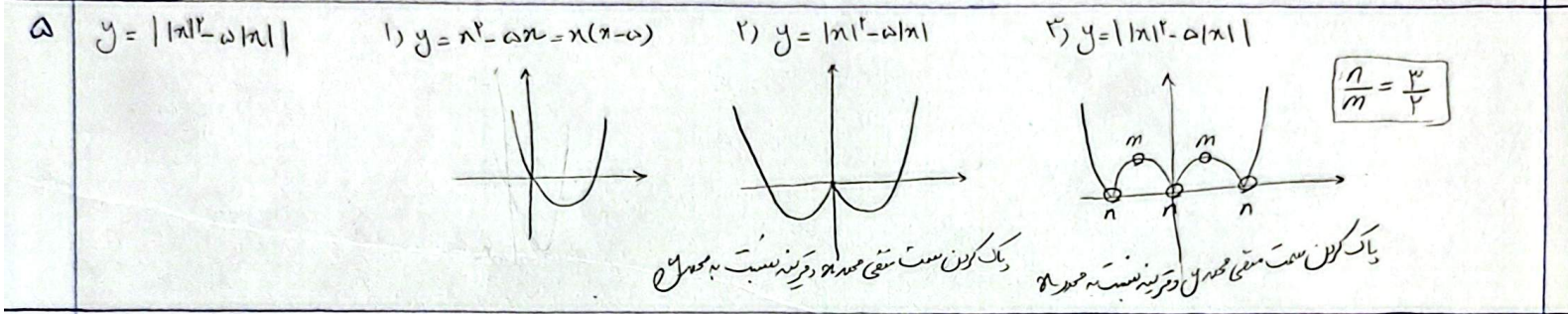
۲  $2ax^2 - ax + 11a = x \Rightarrow 2ax^2 - 4x + 11a = 0 \Rightarrow 3x - f(2a)(11a) = 0 \Rightarrow a^2 = \frac{1}{3} \Rightarrow a = \pm \frac{1}{\sqrt{3}}$   
 $a = \frac{1}{\sqrt{3}} \Rightarrow x^2 - 4x + 9 = (x-3)^2 \Rightarrow x = 3$   $\alpha = -\frac{1}{3} \Rightarrow -x^2 - 2x - 9 = -(x+2)^2 \Rightarrow x = -2$

۳  $y = x^3 - 12x + 2 \Rightarrow y' = 3x^2 - 12$   

x	-2	2
f'	+	-
f	↗	↘

 نقطه سرجی A(2, -14)

۴  $y' = 3x^2 + 2ax - 2b = 3x(x + \frac{2a}{3}) = 3x^2 + 2ax \Rightarrow \begin{cases} a = 3 \\ b = 0 \end{cases} \Rightarrow y = x^3 + 3x^2 - 4$   
 A(0, -4) B(-2, 0)  $|AB| = \sqrt{4+16} = 2\sqrt{5}$



۷  $f(x) = \sqrt[3]{x^2} |x-a| \Rightarrow f(x) = \begin{cases} \sqrt[3]{x^2}(x-a) & x > a \\ \sqrt[3]{x^2}(a-x) & x < a \end{cases} \Rightarrow f'(x) = \begin{cases} \frac{2x}{3\sqrt[3]{x^4}}(x-a) + \sqrt[3]{x^2} & x > a \\ -\frac{2x}{3\sqrt[3]{x^4}}(a-x) - \sqrt[3]{x^2} & x < a \end{cases}$   
 $\frac{2}{3\sqrt[3]{x^4}}(x-a) + \sqrt[3]{x^2} = 0 \Rightarrow 2(x-a) = -3x \Rightarrow 5x = 2a \Rightarrow x = \frac{2}{5}a$   
 بحرانی  $\begin{cases} f(0) = 0 \\ f(\frac{2}{5}a) = \sqrt[3]{\frac{4}{25}a^2} |-\frac{3}{5}a| = \frac{4}{25}a^2 \sqrt[3]{\frac{27}{125}} = \frac{2\sqrt[3]{6}}{125}a^2 \\ f(a) = 0 \end{cases}$   
 $a > 0 \Rightarrow \frac{2\sqrt[3]{6}}{25 \times 125} a^3 = \frac{2\sqrt[3]{6}}{125} a^3 \Rightarrow a^3 = \frac{25 \times 125}{3\sqrt[3]{6}} \Rightarrow a = \frac{25}{\sqrt[3]{6}}$   
 $a < 0 \Rightarrow \frac{-2\sqrt[3]{6}}{25 \times 125} a^3 = \frac{2\sqrt[3]{6}}{125} a^3 \Rightarrow a^3 = -\frac{25}{\sqrt[3]{6}} \Rightarrow a = -\frac{25}{\sqrt[3]{6}}$

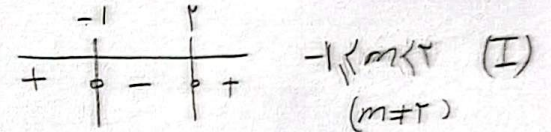
۸  $f(x) = \sqrt{x|x| - x} = \sqrt{x(|x| - 1)} \Rightarrow f(x) = \begin{cases} \sqrt{x^2 - x} & x > 1 \\ \sqrt{-x^2 - x} & -1 < x < 0 \end{cases} \Rightarrow f'(x) = \begin{cases} \frac{2x-1}{2\sqrt{x(x-1)}} & x > 1 \\ \frac{-2x-1}{2\sqrt{-x(x+1)}} & -1 < x < 0 \end{cases}$   

x	-1	0	1
f'	-	+	-
f	↘	↗	↘

 $Df = [-1, 0] \cup [1, +\infty)$   
 $f(x) = 0 \Rightarrow x = \frac{1}{2}$   $\frac{x}{f'} = \frac{1}{2} \Rightarrow x = 1$   
 بحرانی  $\{-1, 0, 1, \frac{1}{2}\} \Rightarrow K = 4$   
 $\frac{K_{m+n}}{K-n} = \frac{f(0)+1}{1-1} = \frac{1}{2}$   
 بحرانی  $m=0$

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$$y = \frac{m x + r}{x - 1 + m} \Rightarrow y' = \frac{m(m-1) - r}{(x+m-1)^2} \Rightarrow \frac{m(m-1) - r}{(x+m-1)^2} < 0 \Rightarrow m(m-1) - r < 0 \Rightarrow m^2 - m - r < 0$$



$$x + m - 1 = 0 \Rightarrow x = 1 - m \Rightarrow 1 - m < 1 \Rightarrow m > 0 \quad (\text{II}) \Rightarrow (\text{I}) \cap (\text{II}) : 0 < m < 2 \Rightarrow \{0, 1\} \quad \boxed{\text{محل جواب}}$$

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$$f(x) = \frac{x}{1 - x|x|} \Rightarrow \begin{cases} x > 0 & \frac{x}{1 - x^2} \Rightarrow \frac{(1 - x^2) - (-2x)(x)}{(1 - x^2)^2} = \frac{x^2 + 1}{(1 - x^2)^2} \\ x < 0 & \frac{x}{1 + x^2} \Rightarrow \frac{(1 + x^2) - (2x)(x)}{(1 + x^2)^2} = \frac{1 - x^2}{(1 + x^2)^2} \end{cases}$$

$$D_f = \mathbb{R} - \{1, -1\}$$

با توجه به اینکه نقطه‌های این نمودار