کسرا امیری نیا

۲۰ آفرین خیلی عالی نوشتی!

①

$[1,3] \to \dfrac{1-\frac{a}{3}-1+a}{3-1} = \dfrac{\frac{2}{3}a}{2} = \dfrac{a}{3}$

$y = 1-\dfrac{a}{x} \to y' = \dfrac{a}{x^2}$

$\to \dfrac{a}{3} = \dfrac{a}{x^2} \to x^2 = 3 \to x = +\sqrt{3}$ ✓ , $x = -\sqrt{3}$ ✗

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②

$y = x$
$y = 2ax^2 - 5x + 18a$ } $\to 2ax^2 - 6x + 18a = 0$

$\Delta = 0 \to 36 - 144a^2 = 0 \to a = \pm\dfrac{1}{2}$

$a = \dfrac{1}{2} \to x^2 - 6x + 9 = 0 \to (x-3)^2 = 0 \to x = 3$ ✗

ناحیه سوم نیست

$a = -\dfrac{1}{2} \to -x^2 - 6x - 9 = 0 \; -(x+3)^2 = 0 \to x = -3$ ✓

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③

$y = x^3 - 12x + 2 \to y' = 3x^2 - 12$

x	$-\infty$	-2	2	$+\infty$
y'	+	−	+	
y	↗	↘	↗	
		18	−14	

که طبق آزمون مشتق اول ۱۴- در نقطه (۲، ۱۴-) مقدار مینیمم نسبی است.

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④

$y = x^3 + ax^2 - 2bx - 4 \to y' = 3x^2 + 2ax - 2b = mx(x+2) = mx^2 + 2mx$

$m = 3$, $a = 3$, $b = 0 \to y = x^3 + 3x^2 - 4 \xrightarrow{x=-2} -8+12-4 = 0$ ماکسیمم نسبی

$x = 0 \to -4$ مینیمم نسبی

x		-2		0	
y'	+		−		+
y	↗		↘		↗
		$(-2,0)$		$(0,-4)$	

$\to \sqrt{4+16} = 2\sqrt{5}$

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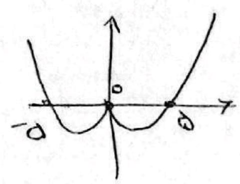
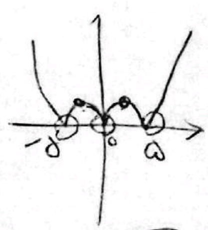
⑤

$f(x) = x^2 - 5|x|$

$f(x) = \begin{cases} x^2 - 5x & x \geq 0 \\ x^2 + 5x & x < 0 \end{cases}$



$\longrightarrow |f(x)|$



$m = 2$
$n = 3$

$\dfrac{n}{m} = \dfrac{3}{2}$

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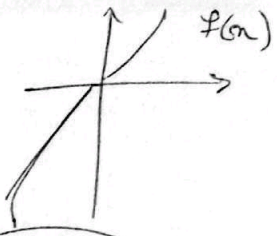
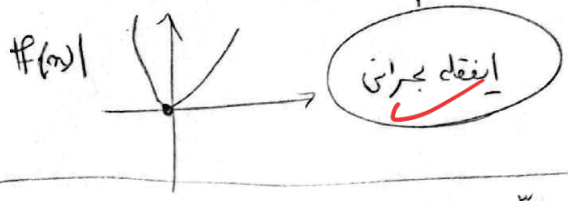
$$f(x)=\begin{cases} x^2+3x & x\geq 0 \\ -x^2+3x & x<0 \end{cases}$$





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$$f(x)=\sqrt[3]{x^2}\,|x-a| \longrightarrow f(x)=-\sqrt[3]{x^2}\,(x-a)$$

$x=0 \quad x=a \quad x\in[0,a]$

$$f'(x)=-\left(\frac{2}{3\sqrt[3]{x}}(x-a)+\sqrt[3]{x^2}\right)=0 \longrightarrow \frac{2(x-a)+3x}{3\sqrt[3]{x}}=0 \longrightarrow x=\frac{2a}{5}$$

$$f\left(\frac{2a}{5}\right)=\sqrt[3]{\frac{4a^2}{25}}\left(\frac{2a}{5}\right)=\frac{4}{9} \longrightarrow \sqrt[3]{\frac{4a^3}{5^3}}=\frac{1}{3} \longrightarrow \left(\frac{a}{5}\right)^3=\frac{1}{5^3}$$

$$\frac{a}{5}=\frac{1}{3} \longrightarrow a=\frac{5}{3}$$

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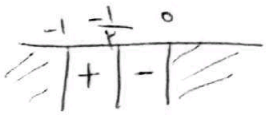
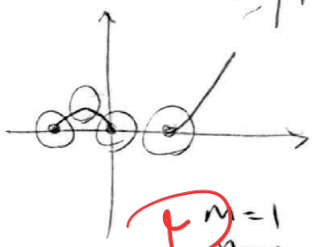
$$f(x)=\sqrt{x|x|-x} \xrightarrow{x\geq 0} f'(x)=\frac{2x-1}{2\sqrt{x^2-x}}=0 \longrightarrow x=\frac{1}{2}$$

$[-1,0]\cup[1,+\infty)$

$$x\leq 0 \longrightarrow f'(x)=\frac{-2x+1}{2\sqrt{-x^2-x}}=0 \longrightarrow x=\frac{1}{2}$$

x	-1		$-\frac{1}{2}$		0		1
f'	$\nexists$	$+$	0	$-$	$\nexists$	$+$	
f	$\times$	$\nearrow$	$\frac{3}{4}$	$\searrow$	$\times$	$\nearrow$	
	0				0		0





$m=1$
$m=0$

$$\frac{0+3}{0-3}=-1$$

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$$y=\frac{mx+2}{x+m-1} \longrightarrow y'=\frac{m^2-m-2}{(x+m-1)^2} \longrightarrow m^2-m-2=0 \rightsquigarrow (m-2)(m+1)=0$$

$m=2$
$m=-1$

	-1		2	
	$+$	$-$	0	$+$

$$2-m\leq 1 \longrightarrow m\geq 0 \longrightarrow 0\leq m<2 \longrightarrow m=0,1$$

دو مقدار

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$$f(x) = \frac{x}{1-x|x|} \xrightarrow{x>0} \frac{x}{1-x^2} \xrightarrow{\text{مشتق}} \frac{1+x^2}{1-x^2}$$

$x=1$ ی

$$\xrightarrow{x<0} \frac{x}{1+x^2} \xrightarrow{\text{مشتق}} \frac{1-x^2}{1+x^2} \longrightarrow x=-1$$

۱ بحرانی

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