کیمیا امیری‌نیا

①

$[1,3] \to \dfrac{1-\frac{a}{3}-1+a}{3-1} = \dfrac{\frac{2}{3}a}{2} = \dfrac{a}{3}$

$y = 1 - \dfrac{a}{x} \to y' = \dfrac{a}{x^2}$

$\to \dfrac{a}{3} = \dfrac{a}{x^2} \to x^2 = 3 \to x = +\sqrt{3}$ ✓ , $x = -\sqrt{3}$ ✗

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②

$\begin{cases} y = x \\ y = 2ax^2 - 5x + 18a \end{cases} \to 2ax^2 - 6x + 18a = 0$

$\Delta = 0 \to 36 - 144a^2 = 0 \to a = \pm\dfrac{1}{2}$

$a = \dfrac{1}{2} \to x^2 - 6x + 9 = 0 \to (x-3)^2 = 0 \to x = 3$ ✗

ناحیه سوم نیست

$a = -\dfrac{1}{2} \to -x^2 - 6x - 9 = 0 \quad -(x+3)^2 = 0 \to x = -3$ ✓

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③

$y = x^3 - 12x + 2 \to y' = 3x^2 - 12$

x	$-\infty$	-2	2	$+\infty$
y'	+	−	+	
y	↗	↘	↗	
		18	−14	

که طبق آزمون مشتق اول ۱۴- در نقطه (۱۴- ,۲) مقدار مینیمم نسبی است.

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④

$y = x^3 + ax^2 - 2bx - 4 \to y' = 3x^2 + 2ax - 2b = mx(x+2) = mx^2 + 2mx$

$m = 3, \; a = 3, \; b = 0 \to y = x^3 + 3x^2 - 4 \xrightarrow{x=-2} -8 + 12 - 4 = (0)$ ماکسیمم نسبی

$x = 0 \to (-4)$ مینیمم نسبی

x		-2		0	
y'	+		−		+
y	↗		↘		↗
		$(-2,0)$		$(0,-4)$	

$\to \sqrt{4+16} = 2\sqrt{5}$

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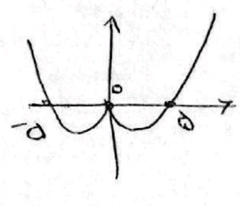
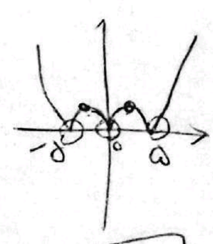
⑤

$f(x) = x^2 - 5|x|$

$f(x) = \begin{cases} x^2 - 5x & x \geq 0 \\ x^2 + 5x & x < 0 \end{cases}$



$\longrightarrow |f(x)|$



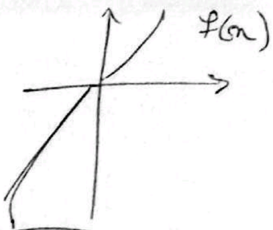
$m = 2$

$n = 3$

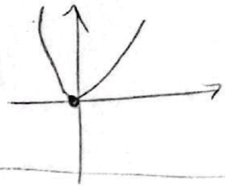
$\dfrac{n}{m} = \dfrac{3}{2}$

(6)

$$f(x) = \begin{cases} x^2 + 3x & x \geq 0 \\ -x^2 + 3x & x < 0 \end{cases}$$



$|f(x)|$



این نقطه بحرانی

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(7)

$f(x) = \sqrt[3]{x^2}\,|x-a| \longrightarrow f(x) = -\sqrt[3]{x^2}\,(x-a)$

$x=0 \quad x=a \quad x \in [0,a]$

$f'(x) = -\left(\left(\frac{2}{3\sqrt[3]{x}}\right)(x-a) + \sqrt[3]{x^2}\right) = 0 \to \frac{2(x-a)+3x}{3\sqrt[3]{x}} = 0 \to x = \frac{2a}{5}$

$f\left(\frac{2a}{5}\right) = -\sqrt[3]{\frac{4a^2}{25}}\left(\frac{-3a}{5}\right) = \frac{3}{2} \to \sqrt[3]{\frac{4a^5}{5^5}} = \frac{1}{2} \to \left(\frac{a}{5}\right)^5 = \frac{1}{32}$

$\frac{a}{5} = \frac{1}{2} \to a = 2.5$

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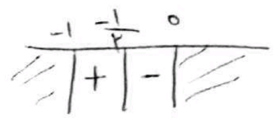
(8)

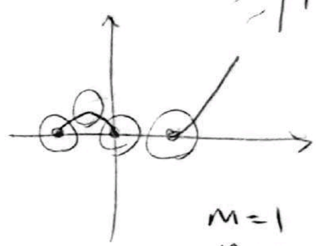
$f(x) = \sqrt{x|x| - x} \xrightarrow{x \geq 0} f'(x) = \frac{2x-1}{2\sqrt{x^2-x}} = 0 \to x = \frac{1}{2}$ ✗

$[-1,0] \cup [1,+\infty)$

$x \leq 0 \quad f'(x) = \frac{-2x+1}{2\sqrt{-x^2-x}} = 0 \to x = \frac{-1}{2}$ ✓

x		-1	$-\frac{1}{2}$	0	1	
f'	ن ت	+	−	ن ت	+	
f	✗	↗	↘	✗	↗	
		0	$\frac{1}{2}$	0	0	





$m=1$
$n=0$
$k=2$

$\frac{4+0}{4-0} = 1$

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(9)

$y = \frac{mx+2}{x+m-1} \to y' = \frac{m^2-m-2}{(x+m-1)^2} \to m^2-m-2=0 \to (m-2)(m+1)=0$

$m=2$
$m=-1$

-1		2	
+	−	+	

$1-m \leq 1 \to m \geq 0 \to 0 \leq m < 2 \to m = 0, 1$

دو مقدار

$$f(x)=\frac{x}{1-x|x|}$$

$x>0$: $\dfrac{x}{1-x^2}$ — مشتق $\Rightarrow \dfrac{1+x^2}{(1-x^2)^2}$

$x=1$ ✗

$x<0$: $\dfrac{x}{1+x^2}$ — مشتق $\dfrac{1-x^2}{(1+x^2)^2} \to x=-1$

-1 بحرانی