

۲. آزمون عالی دوستی!

۱- $f(x) = 1 - \frac{a}{x^3}$ $f(b) - f(a) = \frac{f(b) - f(a)}{b-a}$ $f'(x) = \frac{a}{x^3}$

$f(1) = 1 - a$

$\frac{1 - \frac{a}{x^3} - 1 + a}{x - 1} = \frac{2a}{9} = \frac{a}{3}$ ۲

$f'(x) = \frac{a}{x^3} \rightarrow \frac{a}{3} = \frac{a}{x^3} \rightarrow x = \pm\sqrt{3} \rightarrow \sqrt{3}$ قابل قبول ۲

۲- $y = 2ax^2 - \omega x + 11a$ $2ax^2 - \omega x + 11a = x$ ۲

$\Rightarrow y=x$ (x < 0) $2ax^2 - 4x + 11a = 0$
 $a x^2 - 3x + 9a = 0$

$\Delta = 0 \rightarrow 9 - 36a^2 = 0 \Rightarrow 36a^2 = 9$

$a^2 = \frac{9}{36} \rightarrow a = \pm \frac{3}{6} = \pm \frac{1}{2}$ $\frac{1}{2}$ قابل قبول ۲

$a = \frac{1}{2}$ قابل قبول ۲

۳- $y = x^3 - 12x + 2$ ۳

$y' = 3x^2 - 12$ $x \mid \begin{matrix} -2 & 2 \end{matrix}$ $y' \mid \begin{matrix} + & 0 & - & 0 & + \end{matrix}$ $y \mid \begin{matrix} \nearrow & \searrow & \nearrow \end{matrix}$ $\begin{matrix} \text{بلند} \\ \text{منبسط} \\ \text{نشی} \end{matrix}$

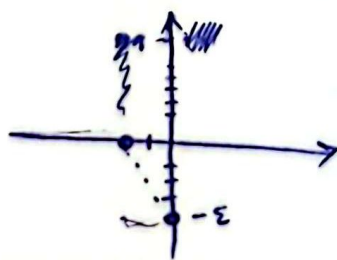
$y = \underbrace{(2^3)}_8 - \underbrace{12(2)}_{-24} + 2 = -14$ ۲

۴- $y = x^3 + ax^2 - 2bx - 4$ ۴

$y' = 3x^2 + 2ax - 2b$ $f'(0) = -2b = 0 \rightarrow b = 0$
 $f'(-2) = 12 - 4a = 0 \rightarrow a = 3$

$f(0) = -4$

$f(-2) = 0$



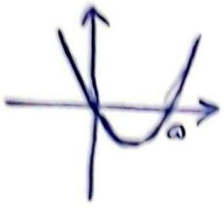
$\Delta = 4 + 12 = 16 \Rightarrow \sqrt{16} = 4$ ۲

$$f(x) = x^r - a|x|$$

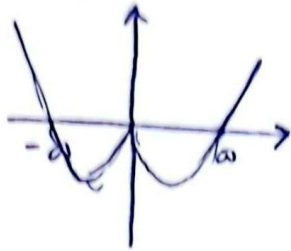
-2

$$y = |f(x)|$$

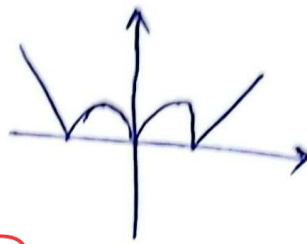
$$y = x^r - ax$$



$$y = x^r - |ax|$$



$$y = |x^r - a|x||$$



$$m = r \rightarrow \frac{n}{m} = \frac{r}{r} = 1, \infty$$

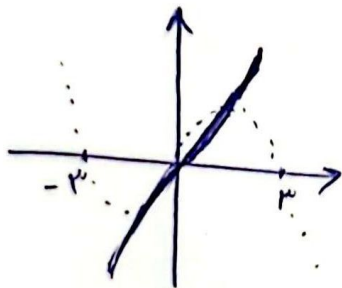
Ⓟ

$$y = |f(x)|$$

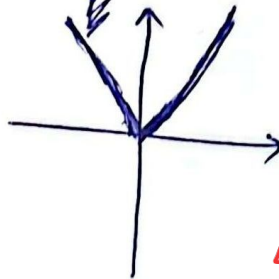
-9

$$f(x) = x(|x| + 3) \rightarrow y = |x(|x| + 3)| \quad \text{تعداد نقاط بحرانی؟}$$

$$f(x) = \begin{cases} x^2 + 3x & x \geq 0 \\ -x^2 + 3x & x < 0 \end{cases}$$



$|f(x)|$



Ⓟ

* یک نقطه بحرانی دارد.

$$f(x) = \sqrt[r]{x^r} (a - x) = ax^{\frac{r-1}{r}} - x^{\frac{r}{r}}$$

$$f'(x) = \frac{r-1}{r} ax^{\frac{1}{r}} - \frac{r}{r} x^{\frac{r-1}{r}} \rightarrow \frac{r-1}{r} x^{\frac{1}{r}} (a - \frac{r}{r-1} x) \rightarrow f'(x) = \frac{r(a - \frac{r}{r-1} x)^{-1}}{r^{\frac{r-1}{r}} \sqrt[r]{x}}$$

$$\text{نقطه بحرانی } x = \frac{r-1}{r} a \mid x = a \mid x = 0$$

$$f\left(\frac{r-1}{r} a\right) = \frac{r-1}{r} \Rightarrow \sqrt[r]{\left(\frac{r-1}{r} a\right)^r} \left(a - \frac{r-1}{r} a\right) = \frac{r-1}{r}$$

$$\Rightarrow \frac{r-1}{r^{\frac{r-1}{r}}} a^{\frac{r-1}{r}} \cdot \frac{r-1}{r} a^{\frac{1}{r}} = \frac{r-1}{r} \rightarrow \frac{r-1}{r^{\frac{r-1}{r}} \cdot r^{\frac{1}{r}}} a = \frac{r-1}{r}$$

$$a = \frac{r}{r-1} \quad \text{Ⓟ}$$

$$f(x) = \begin{cases} \sqrt{x^2 - x} & x \geq 1 \\ \sqrt{-x^2 - x} & x \leq 0 \end{cases}$$

$$f'(x) = \begin{cases} \frac{2x-1}{2\sqrt{x^2-x}} & x \geq 1 \\ \frac{-2x-1}{2\sqrt{-x^2-x}} & x \leq 0 \end{cases}$$

تقریباً $f'(x) = \{1, -1, 0\} \rightarrow k = r$

~~ممكن~~ $f'(x) = 0 \rightarrow \left\{ -\frac{1}{r} \right\}$

$f'(-\frac{1}{r}) > 0$ و $f'(-\frac{1}{r}) < 0 \rightarrow$ $\begin{matrix} \text{delo} \\ \text{Max} \\ \text{نشی} \end{matrix} = -\frac{1}{r}$
 $m = 1$

$\frac{k+m+n}{k-n} \xrightarrow[m=1]{k=r, n=0} \frac{r}{r} = 1$ Ⓟ

$$y' = \frac{m(m-1) - r}{(x-1+m)^2} \Rightarrow \frac{m^2 - m - r}{(x-1+m)^2} \leq 0$$

$\Rightarrow m^2 - m - r \leq 0 \rightarrow -1 \leq m \leq r$
 $m \geq 0$ (تقریباً $(1, +\infty)$) Ⓟ

$\hookrightarrow 0 \leq m \leq r \xrightarrow[m \neq r]{} \boxed{m = 0, 1}$ Ⓟ

$D_{f(x)} = 1 - x|x| \rightarrow x|x| = \begin{cases} x \geq 0 & x^2 = 1 \rightarrow x = 1 \checkmark \\ x \leq 0 & -x^2 = 1 \rightarrow x^2 = -1 \checkmark \end{cases}$ - 10

$D_{f(x)} = \mathbb{R} - \{1\}$

مسئله 2
 $\begin{cases} x > 0 \rightarrow f'(x) = \frac{1-x^2+2x^2}{(1-x^2)^2} = \frac{x^2+1}{(1-x^2)^2} \rightarrow x^2=1 \checkmark \\ x < 0 \rightarrow f'(x) = \frac{1+x^2-2x^2}{(1+x^2)^2} \rightarrow \frac{1-x^2}{(1+x^2)^2} \rightarrow x^2=1 \checkmark \end{cases}$
 کبر فقط بجای Ⓟ