باران ترسلیمی   دوازدهم هنر B

۱-

$$\text{آهنگ تغییرات} = \frac{f(b)-f(a)}{b-a}$$

$$f(3) = 1 - \frac{a}{3}$$

$$f(1) = 1 - a$$

$$\frac{\cancel{1} - \frac{a}{3} - \cancel{1} + a}{2} = \frac{2a}{6} = \frac{a}{3}$$

$$f'(x) = \frac{a}{x^2} \rightarrow \frac{a}{3} = \frac{a}{x^2} \rightarrow x = \pm\sqrt{3} \rightarrow \sqrt{3}$$ قابل قبول

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$$y = 2ax^2 - 5x + 18a \qquad 2ax^2 - 5x + 18a = x$$

معادلهٔ نیمساز ناحیه ۳ $\Rightarrow y = x \quad (x<0)$

$$2ax^2 - 6x + 18a = 0$$

$$ax^2 - 3x + 9a = 0$$

$$\xrightarrow{\Delta = 0} 9 - 36a^2 = 0 \Rightarrow 36a^2 = 9$$

$$a^2 = \frac{9}{36} \rightarrow a = \pm\frac{3}{6} = \frac{1}{2} \xrightarrow{\text{نیمساز ناحیه ۳}}$$

$a = -\frac{1}{2}$ قابل قبول

۳-

$$y = x^3 - 12x + 2$$

$$y' = 3x^2 - 12$$

x		-2		2	
y'	$+$	0	$-$	0	$+$
y	↗		↘		↗

طول مینیمم نسبی $\rightarrow 2$

$$y = \underbrace{(2^3)}_{8} - \underbrace{12(2)}_{-24} + 2 = \boxed{-14}$$

۴-

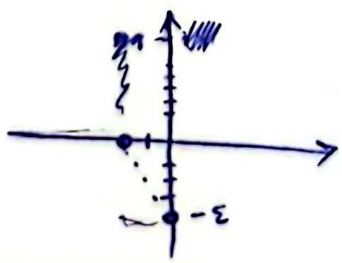
$$y = x^3 + ax^2 - 2bx - 4$$

$$y' = 3x^2 + 2ax - 2b \qquad f'(0) = -2b = 0 \rightarrow b = 0$$

$$f'(-2) = 12 - 4a = 0 \rightarrow a = 3$$

$$f(0) = -4$$

$$f(-2) = 0$$



$$\text{فاصله} = \sqrt{4+16} = 2\sqrt{5}$$

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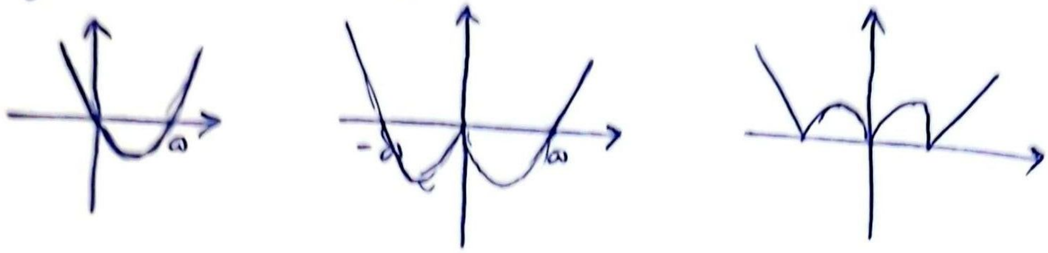
$f(x) = x^2 - 5|x|$

$y = |f(x)|$

$y = x^2 - 5x$ $\qquad$ $y = x^2 - |5x|$ $\qquad$ $y = \left| x^2 - 5|x| \right|$



$$m = 2,\quad n = 3 \rightarrow \frac{n}{m} = \frac{3}{2} = 1{,}5$$

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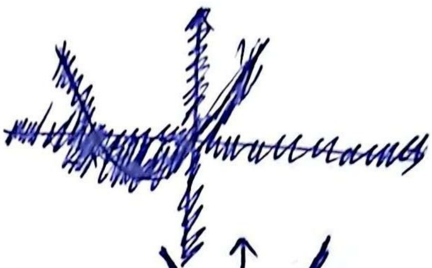
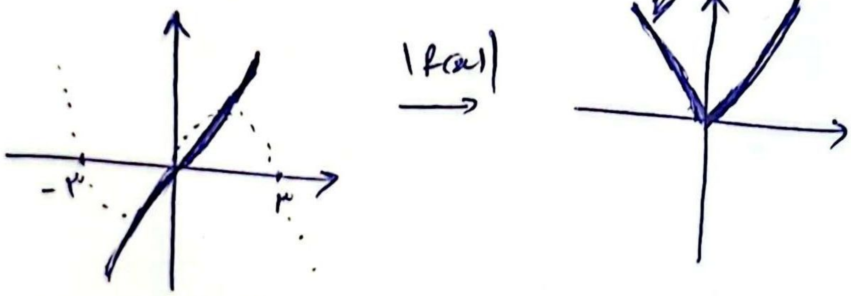
$y = |f(x)|$

$f(x) = x(|x| + 3) \rightarrow y = \left| x(|x|+3) \right|$ تعداد نقاط بحرانی؟

$$f(x) \begin{cases} x^2 + 3x & x \geq 0 \\ -x^2 + 3x & x < 0 \end{cases}$$





* یک نقطه‌ی بحرانی دارد.

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$f(x) = \sqrt[3]{x^2}\,(a - x) = a x^{\frac{2}{3}} - x^{\frac{5}{3}}$

$f'(x) = \frac{2}{3} a x^{-\frac{1}{3}} - \frac{5}{3} x^{\frac{2}{3}} \rightarrow \frac{2}{3} x^{-\frac{1}{3}} \left(a - \frac{5}{2} x\right) \rightarrow f'(x) = \frac{2\left(a - \frac{5}{2}x\right)}{3\sqrt[3]{x}}$

نقطه بحرانی $x = \frac{2}{5} a \;/\; x = a \;/\; x = 0$

$f\left(\frac{2}{5}a\right) = \frac{3}{2} \Rightarrow \sqrt[3]{\left(\frac{2}{5}a\right)^2}\left(a - \frac{2}{5}a\right) = \frac{3}{2}$

$\Rightarrow \frac{4}{25} a^2 \times \frac{27}{125} a^3 = \frac{27}{8} \rightarrow \frac{4a^5}{25 \times 125} = \frac{1}{8}$

$a = \frac{5}{2}$

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$$f(x)=\begin{cases}\sqrt{x^2-x} & x\geq 1\\ \sqrt{-x^2-x} & x\leq 0\end{cases}$$

$$f'(x)=\begin{cases}\dfrac{2x-1}{2\sqrt{x^2-x}} & x\geq 1\\[2mm] \dfrac{-2x-1}{2\sqrt{-x^2-x}} & x\leq 0\end{cases}$$

تعریف نشده $f'(x)=\{1,-1,0\} \longrightarrow k=4$

$[redacted]$ x $f'(x)=0 \to \{\frac{-1}{2}\}$

$f'_-(\frac{-1}{2})>0$ و $f'_+(\frac{-1}{2})<0 \to$ Max نسبی $=\frac{-1}{2}$ طول

$m=1$

$$\frac{km+n}{k-n} \xRightarrow[k=4,\ n=0]{m=1} \frac{4}{4}=1$$

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$$y'=\frac{m(m-1)-2}{(x-1+m)^2} \Rightarrow \frac{m^2-m-2}{(x-1+m)^2}\leq 0$$

$\Rightarrow m^2-m-2\leq 0 \to -1\leq m\leq 2$

$m\geq 0$ $\quad (1,+\infty)$ در منفی دامنه

$\cap,\quad 0\leq m\leq 2 \xrightarrow[m\neq 2]{} \boxed{m=0,1}$

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$D_{f(x)}: 1-x|x| \to x|x| \Rightarrow \begin{cases} x\geq 0 & x^2=1\to x=1 ✓\\ x<0 & -x^2=1, x^2=-1 \text{ ✗}\end{cases}$

$D_{f(x)}=\mathbb{R}-\{1\}$

مشتق $\begin{cases} x>0 \to f'(x)=\dfrac{1-x^2+2x^2}{(1-x^2)^2}=\dfrac{x^2+1}{(1-x^2)^2} \to x^2=-1 \text{ ✗}\quad \text{یک نقطه بحرانی}\\[2mm] x<0 \to f'(x)=\dfrac{1+x^2-2x^2}{(1+x^2)^2} \Rightarrow \dfrac{1-x^2}{(1+x^2)^2} \to x^2=+1 \to x=-1 ✓\end{cases}$