پارسا رندی

سؤال ①

$f(x)=1-\frac{a}{x}$ در بازه $[1,4]$ $\rightarrow \frac{f(4)-f(1)}{4-1}=\frac{1-\frac{a}{4}-(1-a)}{3}=\frac{\frac{3}{4}a}{3}=\frac{a}{4}$

$f'(x)=\frac{a}{x^2} \Rightarrow \frac{a}{x^2}=\frac{a}{4} \Rightarrow x^2=4 \Rightarrow x=\pm\sqrt{4} \Rightarrow \boxed{x=\sqrt{4}}$

سؤال ②

$y=2ax^2-5x+18a \Rightarrow y'=4ax-5 \Rightarrow 4an-5=1 \Rightarrow 4an=6$

نیمساز ۲، ۱ $y=x \rightarrow y'=1 \Rightarrow 2ax=\frac{3}{2}$

$2ax^2-5x+18a=x \Rightarrow 2ax^2-6x+18a=0$

سؤال ③

$y=x^3-12x+2 \quad y'=3x^2-12=0 \Rightarrow 3x^2=12 \Rightarrow x^2=4 \Rightarrow x=\pm 2$

x		-2		$+2$	
y'	$+$	0	$-$	0	$+$
y	↗		↘	min	↗

نقطه $(2,-14) \leftarrow \min$

سؤال ④

$y=x^3+ax^2-2bx-4 \quad y'=3x^2+2ax-2b=3x^2+4x$

$x=0,-2$

$y'(0)=-2b=0 \Rightarrow \boxed{b=0}$

$d=?$

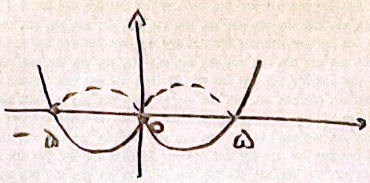
$y'(-2)=12-4a=0 \Rightarrow 4a=12 \Rightarrow \boxed{a=3}$

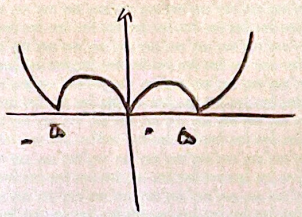
$y=x^3+3x^2-4 \Rightarrow \begin{cases} x=0 \Rightarrow y=-4 \Rightarrow (0,-4) \\ x=-2 \Rightarrow y=0 \Rightarrow (-2,0) \end{cases}$

$d=\sqrt{(-2-0)^2+(0+4)^2}=\sqrt{20}=2\sqrt{5}$

سؤال ۵

$f(x)=x^2-5|x|$ $\begin{cases} x\geq 0 & x^2-5x=x(x-5) \\ x<0 & x^2+5x=x(x+5) \end{cases}$ $x=0,5,-5$



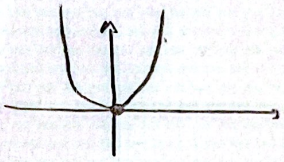


۲ min ش
۲ max ش

$\frac{n}{m}=\frac{2}{2}$

سؤال ۶

$|x(|x|+2)|$ $\begin{cases} x\geq 0 & |x^2+2x|=|x(x+2)| \rightsquigarrow 0,-2 \\ x<0 & |-x^2+2x|=|x(-x+2)| \rightsquigarrow 0,2 \end{cases}$



۳ نقطه

سؤال ۷

$$f(x)=\begin{cases} \sqrt[3]{x^2}\,(x-a) & x\geq a \\ -\sqrt[3]{x^2}\,(x-a) & x\leq a \end{cases} \qquad f'(x)=\begin{cases} \frac{2(x-a)}{3\sqrt[3]{x}}+\sqrt[3]{x^2} & x\geq a \\ -\left(\frac{2(x-a)}{3\sqrt[3]{x}}+\sqrt[3]{x^2}\right) & x\leq a \end{cases}$$

$$\Rightarrow -\left(\frac{2(x-a)}{3\sqrt[3]{x}}+\sqrt[3]{x^2}\right)=0 \Rightarrow -\frac{2(x-a)}{3\sqrt[3]{x}}=\sqrt[3]{x^2} \Rightarrow x-a=-\frac{3}{2}x \Rightarrow x=\frac{2a}{5}$$

$$-\sqrt[3]{\left(\frac{2a}{5}\right)^2}\left(\frac{2a}{5}-a\right)=\frac{3}{2} \Rightarrow \sqrt[3]{\left(\frac{2a}{5}\right)^2}\times a=\frac{5}{2} \Rightarrow \sqrt[3]{\left(\frac{2a}{5}\right)^2}\times\sqrt[3]{a^3}=\frac{5}{2} \Rightarrow \boxed{a=2/5}$$

سؤال ۸

$$f(x)=\begin{cases} \sqrt{x^2-x} & x\geq 1 \\ \sqrt{-x^2-x} & x\leq 0 \end{cases} \Rightarrow f'(x)=\begin{cases} \frac{2x-1}{2\sqrt{x^2-x}} & x\geq 1 \\ \frac{-2x-1}{2\sqrt{-x^2-x}} & x\leq 0 \end{cases}$$

تعریف نشده $f'(x)$ در $\{-1,0,1\}$

$f'(x)=0 \Rightarrow \{-\frac{1}{2}\}$

$f'_-(-\frac{1}{2})>0 \quad f'_+(-\frac{1}{2})<0 \Rightarrow \begin{cases} -\frac{1}{2} \\ m=1 \end{cases} \Rightarrow \frac{km+n}{k-n}=\frac{f(1)+0}{4}=1$

سوال (۱۰)

$f(x) = \frac{x}{1-x|x|}$ $\quad x>0 \quad \frac{x}{1-x^2} \rightsquigarrow y' = \frac{(1-x^2)+2x(x)}{(1-x^2)^2} = \frac{1+x^2}{(1-x^2)^2}$

$x<0 \quad \frac{x}{1+x^2} \rightsquigarrow \frac{1+x^2-2x(x)}{(1+x^2)^2} = \frac{-x^2+1}{(1+x^2)}$

$1-4x^2=0 \longrightarrow 4x^2=1 \Rightarrow x^2=\frac{1}{4} \Rightarrow x=\pm\frac{\sqrt{4}}{4}$ ✓

$-x^2+1=0 \Rightarrow x=\pm 1$ ✓ $\quad$ نقطه بحرانی دارد $(1,-1)$