سینا فوینی دوازدهم سپتامبر ۲۰۱۵

۱-

$f'(x)=\frac{-2x+1}{2\sqrt{-x^2+x}} \rightarrow x=\frac{1}{2}$, $x>0$

$\frac{2x+1}{2\sqrt{x^2+x}}$, $x<0$ $\rightarrow x=-\frac{1}{2} \notin Df$

$Df=$ $0\le x\le 1$, $x\ge 1$, $x\le -1$

$f'(x)$: $+$ | 0 | $-$ | $\frac{1}{2}$ | $+$ | 0 | $-$

$\sqrt{x^2+x}$, 0 , $\sqrt{-x^2+x}$

$\frac{1}{2}$ ← max

min ← نزولی

$x=\frac{1}{2}$ ← بحرانی $0, 1, \pm\frac{1}{2}$

$f'(x)=\frac{1}{2\sqrt{x}}-\frac{1}{\sqrt{a-2x}}=\frac{\sqrt{a-2x}-2\sqrt{x}}{2\sqrt{x}\sqrt{a-2x}}$

۲- $\sqrt{a-2x}=2\sqrt{x}$

$4x=a-2x \Rightarrow x=\frac{a}{6}$

$f(\frac{a}{6})=\sqrt{\frac{a}{6}}+\sqrt{\frac{2a}{3}}=2\sqrt{\frac{a}{6}}$ (max)

$f(0)=\sqrt{a}$

$f(\frac{a}{2})=\sqrt{\frac{a}{2}}+0=\sqrt{\frac{a}{2}}$ (min)

$2\sqrt{\frac{a}{2}}\sqrt{\frac{a}{6}}=\sqrt{12} \rightarrow \frac{a^2}{12}=12 \rightarrow a=\pm 12$

$a=12 \Rightarrow [a]=12$

۳-

$f(x)$

-1 $\quad -\frac{x^2(x^2-1)}{x^2-1}$ $\quad 1$ $\quad \frac{x^2(x^2-1)}{x^2-1} = \frac{x^4-1}{x^2-1}$

$= \frac{-x^4+1+x^2}{x^2-1}$ $\qquad f'(x) = \frac{(4x^3-2x)(x^2-1) - 2x(x^4-1+x^2)}{(x^2-1)^2}$

$= \frac{4x^4-4x^2-2x^2+2x-2x^4+2x^2}{(x^2-1)^2}$ $= \frac{2x^4-4x^2+2x}{(x^2-1)^2}$ $= \frac{2x(x^3-2x+1)}{(x^2-1)^2}$

$x=0$ ↖ x

۴-

$y = 2ax^2+2bx+c \rightarrow$ [illegible]

$y(0)=0 \rightarrow c=0$

$y'(1)=0 \rightarrow 2a+2b=0$

$f(1)=1 \rightarrow a+b+c+d=1 \rightarrow a+b=1$

$f(0)=0 \rightarrow d=0$

$a=+2\ \ b=-2 \rightarrow ab=-4$

۵-

x^2-2x $\quad -\sqrt{2}$ $\quad \sqrt{2}$ $\quad x^2-2x$

$-x^2+2x$ $\qquad -\sqrt{2}<x<\sqrt{2} \rightarrow f(x) = -x^2+2x$

$f'(x) = -2x+2 = 0 \rightarrow x=\pm 1$

$-$ $\quad -1$ $\quad +$ $\quad 1$ $\quad -$

y_{min} $\qquad y_{max}$

$x=-1$

$y=-2$

۶-

$-x^2+2ax+b$ $\qquad x^2+2ax+b$

$y' = \begin{cases} 2x^2+4ax & x>0 \\ -2x^2+4ax & x<0 \end{cases}$

$-2x^2+4ax \xrightarrow[x=-1]{} -2-4a=0 \rightarrow a=-\frac{1}{2}$ $\qquad f(-1)=1 \rightarrow 1-\frac{2}{2}+b=1$

$\frac{b}{a} = -2$ $\qquad b=\frac{2}{2}$

۷-

$y' = 2x+1 \rightarrow x=-\frac{1}{2}$ $\quad (-\frac{1}{2} > \frac{2}{2})$

$\frac{2}{2} \times \frac{1}{4} - \frac{1}{2} + \frac{a}{4} = \frac{2}{2}$

$\frac{a}{a+1} = \frac{2}{2} \rightarrow a=2$

$y = \frac{2x+2}{2x+1} = 0 \rightarrow x = -\frac{2}{2}$

$\frac{}{rx^2+ax+1}$

۸-

$rx\frac{1}{r}-\frac{a}{2}+1=0$ as r

$\frac{b}{a}$ s μ ← طناب ثابت (پیشفرض)

طناب افقی → $\frac{b}{r}=\mu \Rightarrow b=r\mu$

$x=0$ $x=\sqrt{\mu r}$ ۹-

$f'(x)=\frac{rx^{\mu}(x^{\mu}-1)-\mu x^{\mu}(x^{r})}{(x^{\mu}-1)^{\mu}}=\frac{rx^{9}-\mu x^{\mu}-\mu x^{9}}{(x^{\mu}-1)^{\mu}}=\frac{x^{9}-\mu x^{\mu}}{(x^{\mu}-1)^{\mu}}$

$x=\mu$

$\sqrt{\mu r}$ $\quad$ μ $\quad$ 1 $\quad$ 0 $\quad$ $+$

البته آنزل → $[0,\mu]$ → طول μ

$(\mu,\sqrt{\mu r}]$ طول $=\sqrt{\mu r}-\mu$

۱۰-

$f'(x)=\frac{rx^{\mu}(x^{\mu}-\mu)-\mu x(x^{r}-\mu)}{(x^{\mu}-\mu)^{\mu}}=\frac{rx^{\Delta}-1\mu x^{\mu}-\mu x^{\Delta}+9x}{(x^{\mu}-\mu)^{\mu}}$

$=\frac{\mu x^{\Delta}-1\mu x^{\mu}+9x}{(x^{\mu}-\mu)^{\mu}}$

$\mu x(x^{r}-9x^{\mu}+\mu)$

$x=0$

$x=\sqrt{\mu}$

$x=-\sqrt{\mu}$

$x^{r}-9x^{\mu}+\mu=0 \Rightarrow x^{\mu}=t$

$t^{\mu}-9t+\mu=0$ $\quad \frac{9\pm\mu\sqrt{9}}{\mu}=\mu\pm\sqrt{9}$

$x=\sqrt{\mu+\sqrt{9}}\times -\sqrt{\mu+\sqrt{9}}$

$x=\sqrt{\mu-\sqrt{9}}$ $\quad x=-\sqrt{\mu-\sqrt{9}}$

$-\sqrt{\mu}$ $\quad -\sqrt{\mu-\sqrt{9}}$ $\quad 0$ $\quad \sqrt{\mu-\sqrt{9}}$ $\quad \sqrt{\mu}$

$-$ $\quad -$ $\quad +$ $\quad -$ $\quad +$ $\quad +$

البته آنزل $\quad$ البته آنزل

(۳ بازه)