

۱ $f(x) = \sqrt{x(1-x)}$ $x \begin{matrix} -1 & 0 & 1 \\ | & | & | \\ x(1-x) & + & - & + & - \end{matrix}$ $D_f = (-\infty, -1] \cup [0, 1]$

$f(x) = \begin{cases} \sqrt{x(1-x)} & 0 \leq x < 1 \\ \sqrt{x(1-x)} & x < -1 \end{cases}$ $f'(x) = \begin{cases} \frac{1-x}{\sqrt{x-x^2}} & 0 < x < 1 \\ \frac{1+x}{\sqrt{x-x^2}} & x < -1 \end{cases}$ $\Rightarrow f'(x) = 0 \Rightarrow \begin{cases} x = \frac{1}{2} \Rightarrow f = \frac{\sqrt{3}}{2} \\ x = -\frac{1}{2} \Rightarrow f = \frac{\sqrt{3}}{2} \end{cases}$

نقطه بحرانی: $\{-1, 0, 1, \frac{1}{2}\} \Rightarrow K = f \quad m=1 \quad n=0 \Rightarrow K+m, n = 0$

۲ $f(x) = \sqrt{x} + \sqrt{a-2x}$ $f'(x) = \frac{1}{\sqrt{x}} - \frac{2}{\sqrt{a-2x}} = 0 \Rightarrow \frac{1}{\sqrt{x}} = \frac{2}{\sqrt{a-2x}} \Rightarrow \sqrt{x} = \frac{1}{2}\sqrt{a-2x} \Rightarrow 2\sqrt{x} = \sqrt{a-2x} \Rightarrow 4x = a-2x \Rightarrow 6x = a \Rightarrow x = \frac{a}{6}$

$D_f = [0, \frac{a}{2}] \Rightarrow f(0) = \sqrt{a}$ $f(\frac{a}{6}) = \sqrt{\frac{a}{6}} + \sqrt{\frac{2a}{3}} = \sqrt{\frac{a}{6}} + \sqrt{\frac{4a}{3}} = \sqrt{\frac{a}{6}} + 2\sqrt{\frac{a}{3}} = \sqrt{\frac{a}{6}} + \sqrt{\frac{4a}{3}} = \sqrt{\frac{a}{6}} + \sqrt{\frac{8a}{6}} = \sqrt{\frac{9a}{6}} = \sqrt{\frac{3a}{2}} = \sqrt{\frac{3}{2}}\sqrt{a} = y_{max}$

$\sqrt{\frac{a}{6}} + \sqrt{\frac{4a}{3}} = \sqrt{\frac{a}{6}} + \sqrt{\frac{8a}{6}} = \sqrt{\frac{9a}{6}} = \sqrt{\frac{3a}{2}} \Rightarrow a = \frac{2}{3}y^2 \Rightarrow [a] = \frac{2}{3}y^2$

۳ $f(x) = \frac{x^r}{x^r-1} |x^r-1| \Rightarrow f(x) = \begin{cases} \frac{-x^r(x^r-1)}{x^r-1} = \frac{-x^r+x^r}{x^r-1} = \frac{0}{x^r-1} = 0 & -1 < x < 1 \\ \frac{x^r(x^r-1)}{x^r-1} = \frac{x^r-x^r}{x^r-1} = \frac{0}{x^r-1} = 0 & x < -1 \cup x > 1 \end{cases}$

$f'(x) = 0 \Rightarrow \frac{-r x^{r-1} (x^r-1) + x^r (r x^{r-1})}{(x^r-1)^2} = 0 \Rightarrow x = 0$

$f'(x) = \begin{cases} \frac{-r x^{r-1} (x^r-1) + x^r (r x^{r-1})}{(x^r-1)^2} & -1 < x < 1 \\ \frac{r x^{r-1} (x^r-1) + x^r (r x^{r-1})}{(x^r-1)^2} & x < -1 \cup x > 1 \end{cases}$

$\begin{matrix} x & -1 & 0 & 1 & \\ f' & - & + & - & + \\ f & \nearrow & \downarrow & \downarrow & \nearrow \end{matrix}$

۴ $y = ax^2 + bx + c$ $y' = 2ax + b$ $\Rightarrow \begin{cases} 2a(0) + b = 0 \Rightarrow b = 0 \\ 2a(1) + b = 0 \Rightarrow 2a + b = 0 \end{cases} \Rightarrow a = -\frac{b}{2}, b = 2 \Rightarrow a = -1, b = 2 \Rightarrow a+b = -1$

$A: a(0)^2 + b(0) + c = d = 0 \Rightarrow d = 0$

$B: a(1)^2 + b(1) + c = 1 \Rightarrow a + b + c = 1 \Rightarrow a + b = 1 - c = 1 - 0 = 1 \Rightarrow a + b = 1$

۵ $f(x) = \begin{cases} x(3-x) & -\sqrt{3} < x < \sqrt{3} \\ x(x-3) & x > \sqrt{3} \cup x < -\sqrt{3} \end{cases}$ $f'(x) = \begin{cases} 3-2x & -\sqrt{3} < x < \sqrt{3} \\ 2x-3 & x > \sqrt{3} \cup x < -\sqrt{3} \end{cases}$ $f'(x) = 0 \Rightarrow x = \pm 1$

$f(-1) = -\frac{3}{2}(3-\frac{3}{2}) = -\frac{9}{4}$ $f(-1) = -2$ $f(1) = 2$ $f(\sqrt{3}) = 0$

۶ $y = -x^2 + 2ax + b$ $x < 0 \Rightarrow y' = -2x + 2a = 0 \Rightarrow x = a \Rightarrow a = -\frac{1}{2}$ $y = 1 - \frac{1}{4} + b = 1 \Rightarrow b = \frac{3}{4}$ $\frac{b}{a} = -\frac{3}{2}$

۷ $x = \frac{a-1}{a+1}$ $y = \frac{a}{a+1} \Rightarrow (\frac{1-a}{a+1}, \frac{a}{a+1})$ $y' = 2x + 1 = 0 \Rightarrow x = -\frac{1}{2} = \frac{1-a}{a+1} \Rightarrow a = 2$

$\frac{2}{2}(\frac{1}{2})^2 + (-\frac{1}{2}) + \frac{a}{2} = \frac{2}{2} = \frac{a}{2} \Rightarrow \frac{a}{2} = \frac{1}{2} \Rightarrow a = 1$

$y = \frac{2n+r}{n+1} = 0 \Rightarrow n = \frac{r}{2}$

۸ $y = \frac{b}{x} = 2 \Rightarrow b = 2x$ $f(x + \frac{1}{x})^2 = f x^2 + f \frac{1}{x^2} + 1 \Rightarrow a = f \Rightarrow \frac{b}{a} = 2$

۹ $f(x) = \frac{x^e}{x^e-1} \Rightarrow f'(x) = \frac{e x^{e-1} (x^e-1) - x^e (e x^{e-1})}{(x^e-1)^2} = \frac{x^e - 2e x^{2e-1}}{(x^e-1)^2}$

$(\sqrt{3}, \sqrt{3}) \Rightarrow \sqrt{3} - 2 = \sqrt{3} - 2$

$\begin{matrix} x & 0 & 1 & \sqrt{3} \\ f' & + & - & - \\ f & \nearrow & \downarrow & \downarrow \end{matrix}$

۱۰ $f(x) = \frac{x^r-2}{x^r-1} \Rightarrow f'(x) = \frac{r x^{r-1} (x^r-1) - (x^r-2) (r x^{r-1})}{(x^r-1)^2} = \frac{r x^r - r x^{2r-1} + 2r x^{r-1}}{(x^r-1)^2} = \frac{r x (x^r - (x^r-2)) (x^{r-1})}{(x^r-1)^2}$

$\begin{matrix} x & -1 & 0 & 1 & \sqrt{3} \\ f' & + & + & - & - \\ f & \nearrow & \downarrow & \downarrow & \nearrow \end{matrix}$ $\Rightarrow \frac{b}{a} = 3$