

$D_f \Rightarrow x(1-|x|) > 0 \cdot \frac{-1}{+} \frac{-1}{-} \frac{+1}{+} \frac{+1}{-} \rightarrow D_f = (-\infty, -1] \cup [0, 1]$ ①

$f(x) \xrightarrow{[0,1]} \sqrt{x(1+x)} \rightarrow f'(x) = \frac{2x+1}{2\sqrt{x+x^2}} \rightarrow 2x+1=0 \rightarrow x = -\frac{1}{2}$ در بازه نیست
 $f(x) \xrightarrow{(-\infty,-1]} \sqrt{x(1-x)} \rightarrow f'(x) = \frac{-2x+1}{2\sqrt{x-x^2}} \rightarrow -2x+1=0 \rightarrow x = \frac{1}{2}$ در بازه نیست
 جدول علامت: $\begin{matrix} x & -1 & 0 & 1 \\ f' & + & 0 & - \\ f & \nearrow & \searrow & \nearrow \end{matrix}$ $m=1, n=0, k=1 \rightarrow m+n+k=2$

$D_f \Rightarrow x > 0 \cdot \frac{1}{x-2\sqrt{x}} \rightarrow \frac{1}{x} > 2 \rightarrow D_f = [0, \frac{1}{4}]$ ②

$f(x) = \frac{1}{2\sqrt{x}} + \frac{-1}{2\sqrt{1-x}} = \frac{\sqrt{1-x} - \sqrt{x}}{2\sqrt{x}\sqrt{1-x}} \rightarrow \sqrt{1-x} = \sqrt{x} \rightarrow x = \frac{1}{4}$
 $\max \cdot \min = \sqrt{1x} \rightarrow (\sqrt{\frac{1}{4}} + \sqrt{\frac{1}{4}})(\sqrt{\frac{1}{4}}) = \frac{2}{\sqrt{1x}} = \sqrt{1x} - a = \epsilon \rightarrow [a] = \frac{1}{4}$
 جدول علامت: $\begin{matrix} x & 0 & \frac{1}{4} & 1 \\ f' & - & 0 & + \\ f & \searrow & \nearrow & \searrow \end{matrix}$

$D_f = \mathbb{R} - \{\pm 1\}$
 $f(x) \xrightarrow{x < -1} \frac{x^r}{x^r-1} (\epsilon - x^r) \rightarrow f'(x) = \frac{-rx(x^r - rx^r + \epsilon)}{(x^r-1)^2} \rightarrow \Delta < 0$
 $f(x) \xrightarrow{x > 1} \frac{x^r}{x^r-1} (x^r - \epsilon) \rightarrow f'(x) = \frac{+rx(x^r - rx^r + \epsilon)}{(x^r-1)^2}$
 جدول علامت: $\begin{matrix} x & -r & -1 & 0 & +1 & r \\ f' & - & + & 0 & - & + \\ f & \searrow & \nearrow & \searrow & \nearrow & \searrow \end{matrix} \rightarrow \text{نقطه سرج} \rightarrow (0,0)$

$f(x) = ax^r + bx^s + cx + d \quad f'(x) = rax^{r-1} + sbx^{s-1} + c = 0$ ③
 $A(0,0) \xrightarrow{f'} d=0, \quad B(1,1) \xrightarrow{f'} a+b=1 \rightarrow a=-r, b=r \rightarrow ab=-r^2$
 $f' \searrow c=0, \quad f' \searrow r_a + r_b = 0$

$\frac{-\sqrt{r}}{-1} \frac{\sqrt{r}}{+1} \rightarrow f(x) = x(r-x^r) = rx - x^{r+1} \rightarrow f'(x) = r - r x^r \rightarrow f'(x) = 0 \rightarrow x = \pm 1$ ④
 جدول علامت: $\begin{matrix} x & -1 & 0 & 1 & \sqrt{r} \\ f' & - & 0 & + & - \\ f & \searrow & \nearrow & \searrow & \nearrow \end{matrix} \rightarrow \min_{ab} = (-1, -r) \rightarrow \text{نقطه} = -r$

مثلاً $f(x) = x^r + 2ax^r + b$

مثلاً $f(x) = -x^r + 2ax^r + b \xrightarrow{A(-1,1)} 1 + 2a + b = 1 \rightarrow 2a + b = 0 \rightarrow b = -2a$ ⑥

$f'(x) = -rx^{r-1} + 2ax \xrightarrow{f'(a)=0} -r - 2a = 0 \rightarrow a = -\frac{r}{2} \xrightarrow{①} b = \frac{r}{2} \Rightarrow \frac{b}{a} = -1$

A) $\frac{b}{a} = -1 \rightarrow y = \frac{r}{r}(\frac{1}{r}) + (-\frac{1}{r}) + \frac{b}{r} = \frac{r}{r} \rightarrow A(-\frac{1}{r}, \frac{r}{r})$ ⑦

B) $\frac{b}{a} = \frac{r}{r} \rightarrow \frac{a}{a+1} = \frac{r}{r} \rightarrow a = r \Rightarrow y = \frac{rx+r}{r^{n+1}} \xrightarrow{y=0} rx+r=0 \rightarrow x = -\frac{r}{r}$

مثلاً $\frac{b}{a} = \frac{r}{r} = 1 \rightarrow b = r$ $\frac{b}{a} = -1 \rightarrow \frac{r}{r} + (-\frac{r}{r}) + 1 = 0 \rightarrow a = r$ ⑧

$\frac{b}{a} = \frac{r}{r} = 1$

$D_f = \mathbb{R} - \{2\}$ $f(x) = \frac{x^4}{x^2-1} \rightarrow f'(x) = \frac{x^4 - 2x^2}{(x^2-1)^2} \rightarrow x^4 - 2x^2 = 0 \rightarrow x^2(x^2-2) = 0$
 $\rightarrow x=0, \pm\sqrt{2}$
 جدول علامت:

x	0	±√2	±√2	±√2
f	+	+	-	-
f'	+	+	-	-

\rightarrow $\frac{r}{\sqrt{2r-2}} = \sqrt{2r-2} \approx \sqrt{14}$

$f(x) = \frac{x^4 - r}{x^2 - r} \rightarrow f'(x) = \frac{4x^3 - 2rx}{(x^2 - r)^2} \rightarrow 4x^3 - 2rx = 0 \rightarrow 2x(2x^2 - r) = 0$
 $D_f = \mathbb{R} - \{\pm\sqrt{r}\}$

x	-√r	0	±√r	±√r	±√r
f	-	-	+	+	-
f'	-	-	+	+	-

مثلاً $\frac{r}{\sqrt{r}} = \sqrt{r} \approx \sqrt{14}$