

۲۷ Calculus

ب. نام صفا

مستخرجی
داده هم از مشتق B

$x > 0 \Rightarrow f(x) = \sqrt{x(1-x)} \rightarrow f'(x) = \frac{1-2x}{2\sqrt{x-x^2}} \Rightarrow x=0$ (1)
 $x \leq 0$ DF: $[0, 1]$ نقاط $x=0$
 $x=1$

$\rightarrow f(x) = \sqrt{x(1+x)} \rightarrow f'(x) = \frac{1+x}{2\sqrt{x+x^2}}$
 DF: $x \geq 0 \vee x \leq -1$
 نقاط $x=0, x=-1$

$m=1$
 $n=0$
 $K=3$ (منزوی، مشتق وید دین)
 } \rightarrow (C)

$f(x) = \sqrt{x} + \sqrt{a-2x}$ DF: $x \geq 0, a-2x \geq 0$
 $2x \leq a \Rightarrow x \leq \frac{a}{2}$ (2)

$f'(x) = \frac{1}{2\sqrt{x}} + \frac{-1}{\sqrt{a-2x}} \Rightarrow f'(x)=0 \Rightarrow \frac{1}{\sqrt{a-2x}} = \frac{1}{2\sqrt{x}} \Rightarrow 2x = a-2x \Rightarrow 4x = a \Rightarrow x = \frac{a}{4}$

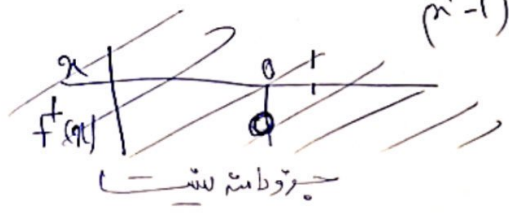
$x=0 \rightarrow f(x) = \sqrt{a}$
 $x = \frac{a}{4} \rightarrow f(x) = \sqrt{\frac{a}{4}}$

$x = \frac{a}{4} \rightarrow f(x) = \sqrt{\frac{a}{4}} + \sqrt{\frac{3a}{4}} = \frac{\sqrt{a} + \sqrt{3a}}{2} = \frac{\sqrt{a}(1+\sqrt{3})}{2} = \frac{\sqrt{4} \sqrt{a}}{2} = \sqrt{a}$
 $\frac{\sqrt{4} \sqrt{a}}{2} \times \sqrt{\frac{a}{4}} = \sqrt{12} \Rightarrow \sqrt{\frac{4}{2} \times \frac{a^2}{4}} = \sqrt{12} \Rightarrow \frac{a^2}{2} = 12 \Rightarrow a^2 = 24 \Rightarrow a = \sqrt{24}$

$x \geq 2 \vee x \leq -2$

$\nabla x \geq 2 \rightarrow f(x) = \frac{x^2}{x^2-1} (x^2-2) \Rightarrow f'(x) = \frac{(2x^2-2)(x^2-1) - (x^2-2)(2x)}{(x^2-1)^2}$ (3)

$f'(x)=0 \Rightarrow (2x^2-2)(x^2-1) = 2x(x^2-2)$
 $2x^2 - 2 - 2x^4 + 2x^2 = 2x^3 - 4x$
 $2x^2 - 2 - 2x^4 + 2x^2 - 2x^3 + 4x = 0$
 $2x^2 - 2 - 2x^4 - 2x^3 + 4x = 0$
 $x(2x^2 - 2 - 2x^3 - 2x^2 + 4) = 0$
 $x=0$

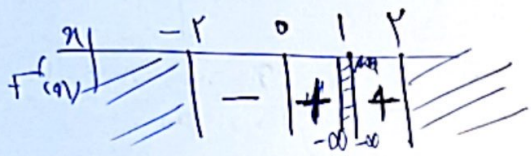


if $x^r \leq \epsilon, -r \leq x \leq r$ $f(x) = \frac{x^r}{x^r - 1} (-x^r + r) = \frac{-x^r + \epsilon x^r}{x^r - 1}$

$f'(x) = \frac{(-\epsilon x^r + r)(x^r - 1) - (x^r)(r - \epsilon x^r)}{(x^r - 1)^2} \Rightarrow f'(x) = 0$

~~$-\epsilon x^r + \epsilon x^r + r x^r - r x = -r x^r + r x$~~

$r x^r - \epsilon x^r + r x = 0 \Rightarrow x = 0$
 $x(r x^{r-1} - \epsilon x^{r-1} + r)$
 $r x (x^r - \epsilon x^r + r)$



نقطه است (0,0) که

$y = ax^r + bx^r + cx + d = f(x) \Rightarrow f'(x) = rax^{r-1} + rbx^r + c \Rightarrow f'(0) = 0 \Rightarrow$ C = 0

مثلاً A: (0,0), B: (1,1)

نقطه است (0,0) در صورتی که

$f(0) = 0 \Rightarrow$ d = 0

$f'(1) = 0 \Rightarrow$ ra + rb = 0 I

$f(1) = 1 \Rightarrow$ a + b = 1 II

I, II \Rightarrow $\begin{cases} ra + rb = 0 \\ ra + rb = r \end{cases} \Rightarrow \begin{cases} a = -r \\ b = r \end{cases}$

$ab = -r^2$

$f(x) = x |r - x^r|$
 $[-1, 0] \cup [r, \sqrt{r}]$

if $x > \sqrt{r} \Rightarrow x < -\sqrt{r} \Rightarrow f(x) = x^r - r^r x \Rightarrow f'(x) = r x^{r-1} - r^r$
 if $-\sqrt{r} \leq x \leq \sqrt{r} \Rightarrow f(x) = r^r x - x^r \Rightarrow f'(x) = r^r - r x^{r-1}$

نقاط بحرانی
 $x = 1 \rightarrow f(x) = r$
 $x = -1 \rightarrow f(x) = -r$
 $x = \sqrt{r} \rightarrow f(x) = 0$
 $x = -\sqrt{r} \rightarrow f(x) = \frac{-r}{r}$

\Rightarrow $f(x) = -r$
 جایی که min

$y = x^r |ax + rax^r + b| \xrightarrow{A(-b/r)} y = -x^r + rax^r + b \Rightarrow y' = -r x^{r-1} + 4ax$

$y'(-1) = 0 \Rightarrow y' = -r - 4a = 0 \Rightarrow 4a = -r \Rightarrow$ a = $-\frac{r}{4}$

\Rightarrow b = $-\frac{r}{4}$

$A(1,1) \Rightarrow y = 1 = 1 + \frac{-r}{4} + b \Rightarrow$ b = $\frac{r}{4}$

$$y = \frac{f(x+3)}{(a+1)x + (a-1)}$$

$$y = \frac{3}{x} x^2 + x + \frac{a}{9} \Rightarrow y' = 2x + 1 \stackrel{y=0}{\Rightarrow} x = -\frac{1}{2} \Rightarrow y = \frac{3}{-1/2} \Rightarrow \frac{3}{f(x)} \Rightarrow -\frac{1}{3} +$$

min نکتہ : $A\left(\frac{-1}{2}, \frac{3}{2}\right)$

مخالف علامت : $(a+1)x = 1-a \Rightarrow x = \frac{1-a}{1+a} \Rightarrow \boxed{a=2}$

مخالف علامت : $y = \frac{a}{a+1}$

$$y = \frac{2x+3}{3x+1} = 0 \Rightarrow \boxed{x = -\frac{3}{2}}$$

$$y = \frac{bx^2 + v}{x^2 + ax + 1}$$

مخالف علامت : $y = \frac{b}{c} = 3 \Rightarrow \boxed{b=12}$

مخالف علامت : $x^2 + ax + 1 = 0$

$$x^2 + ax + 1 = (x+1)^2$$

$$a = -2$$

$$\Rightarrow \frac{b}{a} = \frac{12}{-2} = \boxed{-6}$$

$$f(x) = \frac{x^6}{x^3-1}$$

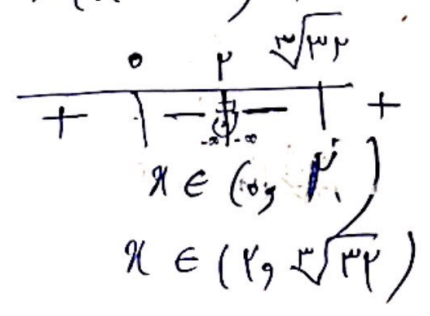
اکثر/کمتر $\rightarrow f'(x) < 0$

$$f'(x) = \frac{6x^5(x^3-1) - (3x^2)(x^6)}{(x^3-1)^2} < 0 \Rightarrow 6x^4 - 3x^2 < 3x^4$$

$$x^4 < 3x^2 \Rightarrow x^2 < 3x \Rightarrow x^2 - 3x < 0$$

$$x^2(x-3) < 0$$

منقسم طرز سے : $\sqrt{3} - 2$



(1.)

$$f(x) = \frac{x^3 - 1}{x^2 - 1}$$

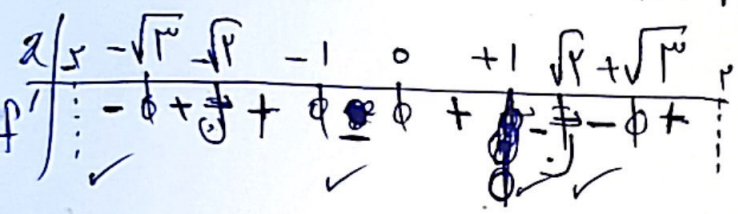
$$f'(x) = \frac{(3x^2)(x^2 - 1) - (x^3 - 1)(2x)}{(x^2 - 1)^2} = 0 \Rightarrow 3x^4 - 12x^2 = 2x^4 - 4x$$

$$2x^4 - 12x^2 + 4x = 0$$

$$x(2x^3 - 12x^2 + 4) = 0$$

$$x(x^3 - 4x^2 + 2) = 0$$

$$f(x) = \frac{3x^2(x^2 - 1) - (x^3 - 1)(2x)}{(x^2 - 1)^2} = 0$$



علامه

$$3x^4 - 12x^2 = 2x^4 - 4x$$

$$2x^4 - 12x^2 + 4x = 0$$

$$2x(x^3 - 4x^2 + 2) = 0$$

$$2x(x^2 - 1)(x^2 - 3) = 0$$