نیلوحیدری — کلاس (۲۷) — جزوه ریاضی ۱۴,۵

سوال ۱

$\lim_{x\to 0} \frac{f(x)}{x} = 0 \rightarrow f(0)=0 \quad \cos^3(2x)+ax^2+b=0$

$1+b=0 \rightarrow b=-1$ (۱,۵)

$\lim_{x\to 0} \frac{f'(x)}{x} \xrightarrow{Hop} \lim \frac{f''(x)}{1} = 2 \rightarrow f''(x)=2$

$f'(x) = 3\cos^2(2x)(2)(-\sin(2x)) + 2ax \rightarrow 6\cos^2(2x)(-\sin(2x))+2ax$

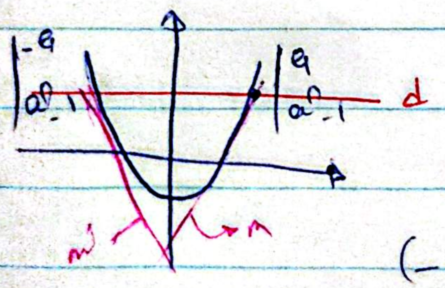
$f''(x)\ 12\cos(2x)(-\sin(2x))(-\sin 2x) + (-\cos(2x))(6\cos^2(2x)) + 2a$

$\rightarrow 12(1)(-0)(-0) + (-1)(6) + 2a = 2$

$2a-6=2 \rightarrow 2a=8 \rightarrow a=4$

$a+b = 4-1 = 3$

سوال ۲



$m_d = 0$

$m \cdot m' = -1 \rightarrow m = 2x = 2a$

$m' \rightarrow 2x \rightarrow -2a$

$(-2a)(2a) = -1 \rightarrow 4a^2 = -1 \rightarrow a^2 = +\frac{1}{4}$ (۱,۵)

$a = \pm\frac{1}{2} \longrightarrow y = a^2 - 1 \Rightarrow \frac{1}{4} - 1 = -\frac{3}{4}$

سوال ۳

$f'(x) = \frac{-2a}{(2x-1)^2} \quad m_d = \frac{\Delta y}{\Delta x} = \frac{6-(-12)}{2,5-(-0,5)} = \frac{18}{3} = 6$

$\frac{-2a}{(2(-0,5)-1)^2} = 6 \rightarrow \frac{-2a}{4} = 6 \rightarrow a = -12$ (۱,۵)

$f(a) = \frac{-12}{-20-1} \rightarrow \frac{-12}{a} = -\frac{4}{3}$

Persian Note Book

Subject:

Date:

سؤال ۴

$y' = 2$, $f(x) = \dfrac{x+9}{ax+1} \rightarrow f'(x) = \dfrac{ax+1-ax-9a}{(ax+1)^2}$

$\xrightarrow{x=1} 2 = \dfrac{(1-a)(1+a)}{(1+a)^2} \rightarrow 2+2a = 1-a \rightarrow 3a = -1 \rightarrow a = -\dfrac{1}{3}$

$y(1) \Rightarrow \dfrac{1-\frac{1}{3}}{-\frac{1}{3}+1} = 1 \Rightarrow 1 = 2+b \rightarrow b = -1$

$a-b \rightarrow -\dfrac{1}{3} - (-1) = ? \rightarrow ? = \dfrac{2}{3}$

۲

سؤال ۵

$g(x) = f(x) \Rightarrow \dfrac{3}{4}\sin x = \sin x + \dfrac{1}{4}\cos x \rightarrow \sin x = \cos x$

$0 < x < \pi \rightarrow x = \dfrac{\pi}{4}$

$f'(x) \Rightarrow \cos x - \dfrac{1}{4}\sin x \xrightarrow{x=\frac{\pi}{4}} \dfrac{2\sqrt{2}}{4} - \dfrac{\sqrt{2}}{4} \rightarrow \dfrac{\sqrt{2}}{4} = m$

$y = \dfrac{\sqrt{2}}{4}x + b \rightarrow \left(\dfrac{\pi}{4}, \dfrac{3\sqrt{2}}{4}\right) \rightarrow y - \dfrac{3\sqrt{2}}{4} = \dfrac{\sqrt{2}}{4}\left(x - \dfrac{\pi}{4}\right)$

$y = 0 \rightarrow -\dfrac{3\sqrt{2}}{4} = \dfrac{\sqrt{2}}{4}\left(x - \dfrac{\pi}{4}\right) \rightarrow -3 = x - \dfrac{\pi}{4}$

$x = -3 + \dfrac{\pi}{4}$

۲

سؤال ۶

$f'(x) = 0 \Rightarrow 9x^2 - 9x - 18 = 0 \quad x^2 - x - 2 = 0 \rightarrow (x-2)(x+1) = 0$

$A = (2, -19) \quad B = (-1, 8) \rightarrow m_{AB} = \dfrac{8-(-19)}{-1-2} \rightarrow \dfrac{27}{-3} = -9$

۲

$9x^2 - 9x - 12 = -9 \rightarrow 3x^2 - 3x - 4 = -3 \rightarrow 3x^2 - 3x - 1 = 0$

$x^2 - 4x - 12 = 0 \quad \Delta = 4 + 56 = 60 \rightarrow \Delta > 0 \rightarrow$ دو ریشه $\rightarrow$ دو نقطه

۱

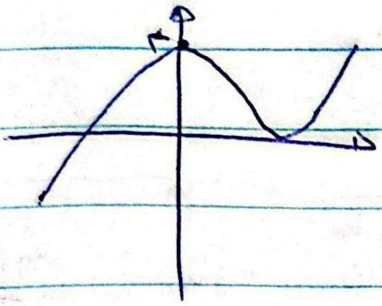
سوال ۸

$A(-1,-4) \xrightarrow{\text{نقطه عطف}} \frac{-a}{3} = -1 \rightarrow \boxed{a=3}$

$-1 + a - b - 1 = -1 + 3 - b - 2 = -4 \rightarrow \boxed{b=5}$

$\boxed{\frac{a}{b} = \frac{3}{5}}$ ✓ ۲

سوال ۹



$f(x) = x^3 + ax^2 + bx + c$

$f(0) = 4 \rightarrow \boxed{c=4}$

$f'(x) = 3x^2 + 2ax + b = 0$

$f'(0) = 0 \rightarrow \boxed{b=0}$

$x(3x + 2a) \rightarrow x = 0$

$\quad \rightarrow x = -\frac{2a}{3}$

$f\left(-\frac{2a}{3}\right) = -\frac{8a^3}{27} + \frac{12a^3}{27} + 0 + 4 \rightarrow \frac{4a^3}{27} + 4 = 0 \rightarrow \boxed{a=-3}$ ۲

$x = \frac{-2(-3)}{3} \rightarrow \boxed{x=2}$ ✓

سوال ۱۰

$f(x) = 4x^3 - 12x = 0 \quad x(x^2 - 3)$

$-\sqrt{3}$	0	$\sqrt{3}$
$-$	$+$	$-$	$+$

$A = (\sqrt{3}, -4),\ B = (-\sqrt{3}, -4)$

$f'(x) = 12x^2 - 12 \Rightarrow$

-1	1
$+$	$-$	$+$

$C = (1, 0)\ D = (-1, 0)$

$m_{AB} = \frac{\Delta y}{\Delta x} = \frac{0}{2\sqrt{3}} = 0 \qquad m'_{CD} = \frac{0}{2} = 0$ ۲

$\tan\alpha = \left|\frac{m - m'}{1 + mm'}\right| \rightarrow 0 \rightarrow \boxed{\alpha = 0 \text{ یا } 180}$ ✓

—۱

$$\lim_{x\to 0^+}\frac{f(x)}{x}=0 \to \lim_{x\to 0^+}\frac{\cos^3(2x)+ax^2+b}{x}=0 \to \lim_{x\to 0^+}\frac{1+b}{x}=0$$

$\hookrightarrow$ $\boxed{b=-1}$

$$\lim_{x\to 0^-}\frac{f'(x)}{x}=2=\lim\frac{-6\sin(2x)\cos^2(2x)+2ax}{x}=2 \xrightarrow{\text{هم‌ارزی}}$$

$$\lim_{x\to 0^-}\frac{(-6\times 2x)+2ax}{x}=2 \to 2a-12=2 \to 2a=14 \to \boxed{a=7}$$

$a+b=7-1=6$

—۳

$$m=\frac{9-(-12)}{2{,}5-(-1{,}5)}=\frac{18}{3}=6 \to y=6x-9$$

$$\frac{a}{2x-1}=6x-9 \to 12x^2-24x+9-a=0 \xrightarrow{\Delta=0} 12^2-4(12)(9-a)=0 \to 12-9+a=0$$

$\hookrightarrow a=-3$

$$f(5)=\frac{-3}{2(5)-1}=\frac{-3}{9}=-\frac{1}{3}$$

—۲

$f(x)=x^2-1 \to f'(x)=2x$

نقاط (α,α^2-1), $(-\alpha,\alpha^2-1)$ را در نظر می‌گیریم

در این صورت خواهیم داشت:

$$f'(\alpha)\times f'(-\alpha)=-1 \to 2\alpha\times(-2\alpha)=-1 \to \alpha^2=\frac{1}{4}$$

$$\hookrightarrow \alpha=\pm\frac{1}{2} \to f(\tfrac{1}{2})+f(-\tfrac{1}{2})=\frac{1}{4}-1+\frac{1}{4}-1=2-0{,}5=1{,}5$$

—۷

$$y'=3kx^2+2(k+1)x \to y''=6kx+2(k+1)=0 \to x=\frac{k+1}{-3k}$$

نقطهٔ عطف در ناحیهٔ دوم است پس ←

(1)
$$\frac{-(k+1)}{3k}<0 \to \frac{-1 \quad 0}{-\ \ \ +\ \ \ -} \to k<-1 \text{ یا } k>0$$

(2)
$$-\frac{(k+1)}{3k}(k)+(k+1)>0 \to \frac{-(k+1)}{3}+k+1>0 \to \frac{2k+2}{3}>0 \to k>-1$$

$1\cap 2 \to k>0$

به ازای هیچ مقدار k منفی و صحیح جواب ندارد!