

(سوال ۱)

$$f(x) = \cos^3(x) + ax^2 + b$$

$$\lim_{x \rightarrow 0^+} \frac{f(x)}{x} \text{ stop} \rightarrow \lim_{x \rightarrow 0^+} \frac{-4\sin(x)\cos^2(x) + 2ax}{1} = 0$$

$$\cos^2(0) + b = 0 \rightarrow b = -1 \rightarrow a + b = 4$$

$$\lim_{x \rightarrow 0^+} \frac{f'(x)}{x} \text{ stop} \rightarrow \lim_{x \rightarrow 0^+} \frac{-12\cos^3(x) + 2a}{1} = 2 \xrightarrow{x=0} -12 + 2a = 2 \rightarrow a = 7$$

$$f(x) = x^2 - 1 \rightarrow f'(x) = 2x$$

(سوال ۲) چون کسی قرینه است پس بی از نقاط a و دیگری را -a در نظری می بینیم

$$(2a) \times (-2a) = 1 \rightarrow -4a^2 = 1 \rightarrow a = \pm \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{4} - 1 = -\frac{3}{4} \quad \text{مجموع عرض نقطه} = 2x = \frac{3}{2} = -1.5$$

(سوال ۳)

$$f(x) = \frac{a}{2x-1} \rightarrow \frac{a}{2x-1} = 4x-9 \rightarrow a = 12x^2 - 22x + 9 \rightarrow 12x^2 - 22x + 9 - a = 0 \xrightarrow{\Delta=0} 54 - 4a(9-a) = 0 \rightarrow a = -3$$

$$\text{intercept} \rightarrow m = \frac{9 - (-3)}{2(2) - (-1)} = 4 \rightarrow y = 4x - 9 \quad / \quad f(2) = \frac{-3}{10-1} = -\frac{1}{3}$$

(سوال ۴)

$$f(x) = \frac{x+a}{ax+1} \rightarrow f'(x) = \frac{1-a^2}{(ax+1)^2} \rightarrow f'(1) = \frac{1-a^2}{(a+1)^2} = 2 \rightarrow \frac{(1-a)(1+a)}{(a+1)^2} = 2 \rightarrow 1-a = 2a+2 \rightarrow 3a = -1 \rightarrow a = -\frac{1}{3}$$

$$f(1) = \frac{1+a}{a+1} = 1 \rightarrow y = 2x + b \xrightarrow{x=1} 2+b=1 \rightarrow b=-1 \Rightarrow a-b = \frac{2}{3}$$

(سوال ۵)

$$f(x) = \sin x + \frac{1}{p} \cos x \quad g(x) = \frac{m}{p} \sin x \rightarrow f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{p} + \frac{\sqrt{2}}{p} = \frac{2\sqrt{2}}{p}$$

$$f(x) = g(x) \rightarrow \sin x + \frac{1}{p} \cos x = \frac{m}{p} \sin x \rightarrow \frac{1}{p} \cos x = \frac{m-1}{p} \sin x \rightarrow \cos x = (m-1) \sin x \quad [0, \pi] \rightarrow x = \frac{\pi}{4}$$

$$f'(x) = \cos x - \frac{1}{p} \sin x \rightarrow f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{p} - \frac{\sqrt{2}}{p} = \frac{\sqrt{2}}{p} \rightarrow \text{intercept} \rightarrow y = \frac{\sqrt{2}}{p}x + b \xrightarrow{x=\frac{\pi}{4}} \frac{\pi\sqrt{2}}{4} + b = \frac{3\sqrt{2}}{4} \rightarrow b = \frac{(14-\pi)\sqrt{2}}{4}$$

$$\frac{\sqrt{2}}{p}x + \frac{(14-\pi)\sqrt{2}}{4} = 0 \rightarrow \frac{\sqrt{2}}{p}x = -\frac{\sqrt{2}(14-\pi)}{4} \rightarrow x = \frac{\pi-14}{p}$$

(سوال ۶)

$$f(x) = 2x^3 - 3x^2 - 12x + 1 \rightarrow f'(x) = 6x^2 - 6x - 12 = 0 \quad \left\{ \begin{array}{l} x=1 \\ x=2 \end{array} \right.$$

$$f'(x) = -9 \rightarrow 6x^2 - 6x - 12 = -9 \rightarrow 6x^2 - 6x - 3 = 0 \xrightarrow{\Delta=0} \text{نقطه چولدار}$$

$$\left. \begin{array}{l} f(-1) = 1 \\ f(2) = -19 \end{array} \right\} \rightarrow m_{AB} = \frac{-19-1}{2-(-1)} = \frac{-20}{3} = -9$$

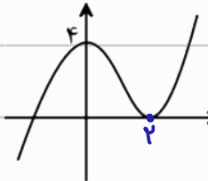
(سوال ۷)

$$f(x) = kx^2 + (k+1)x^2$$

(سوال ۸)

$$f(x) = x^2 + ax^2 + bx - 1 \rightarrow f(-1) = -1 + a - b - 1 = -2 \rightarrow a - b = -2 \rightarrow 3 - b = -2 \rightarrow b = 5$$

$$\rightarrow \frac{a}{b} = \frac{3}{5} \quad \frac{-a}{b} = -1 \rightarrow a = 3 \quad \text{از آنجایی که نقطه (-1, -2) نقطه نصف است}$$



$$f(x) = x^3 + ax^2 + bx + c \quad x=0 \rightarrow c=f \rightarrow f(x) = x^3 + ax^2 + f$$

(سوال 9)

$$f'(x) = 3x^2 + 2ax + b \rightarrow f'(0) = 0 \rightarrow b = 0$$

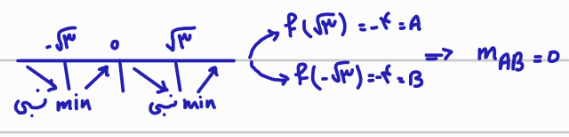
$$\rightarrow f\left(-\frac{2a}{3}\right) = 0 \rightarrow \frac{-2a^3}{27} + \frac{4a^3}{9} + f = 0 \rightarrow \frac{2a^3}{27} + f = 0 \rightarrow a = -3$$

$$\hookrightarrow 3x^2 + 2ax = 0 \rightarrow x(3x + 2a) = 0 \rightarrow 3x + 2a = 0 \rightarrow 3x = -2a \rightarrow x = -\frac{2a}{3} \rightarrow x = 2$$

$$f(x) = x^4 - 4x^2 + 8$$

(سوال 10)

$$f'(x) = 4x^3 - 8x \rightarrow 4x(x^2 - 2) = 0$$



$$f(\sqrt{2}) = -f = A$$

$$f(-\sqrt{2}) = -f = B \Rightarrow m_{AB} = 0$$

\Rightarrow دو خط موازی هستند و زاویه ای با هم نمی سازند

$$f''(x) = 12x^2 - 8 = 0 \rightarrow x = \pm 1$$

$$\left. \begin{array}{l} f(1) = 0 = C \\ f(-1) = 0 = D \end{array} \right\} \rightarrow m_{CD} = 0$$