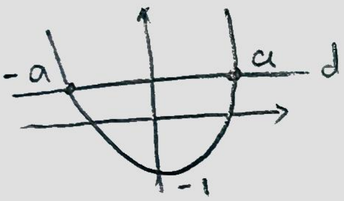
پارمیس بنی‌زاده ، دوازدهم دختر B   ۱۸   تکلیف شماره ۱۷

۱) $\lim \frac{f(x)}{x} = 0 \rightarrow \cos^3(0) + b = 0 \quad b = -1$

$\lim_{x\to 0} \frac{f'(x)}{x} = \frac{-9\sin^2(2x) + 2ax}{x} \xrightarrow{hop} \frac{-24\cos(2x) + 2a}{1}$

$\Rightarrow -24 + 2a = 2 \quad a = 13 \quad a + b = 12$ ۱٫۵

۲) 

$f = x^2 - 1 \quad f' = 2x$

$(2\alpha)(-2\alpha) = -1 \quad \alpha = \frac{1}{2}$

$\alpha \rightarrow \left(\frac{1}{2}, \frac{-3}{4}\right) \quad -\alpha \rightarrow \left(\frac{-1}{2}, \frac{-3}{4}\right)$

مجموع عرض‌ها $\rightarrow 2 \times \frac{-3}{4} = \frac{-3}{2}$ ۲

۳) $f' = \frac{\cancel{4}a}{(2x-1)^2} = \cancel{4}3 \rightarrow a = 12x^2 - 12x + 3$ ۱٫۵

$f(x) = \frac{12x^2 - 12x + 3}{2x - 1} \quad f(5) = \frac{300 - 60 + 3}{9} = \frac{243}{9} = 27$

۴) $f'(1) = 2 \rightarrow \frac{(1)(a+1) - a(1+a)}{(a+1)^2} = \frac{1-a}{a+1} = 2$

$1 - a = 2a + 2 \rightarrow 3a = -1 \quad a = \frac{-1}{3}$

$f(1) = 2 + b = \frac{1 - \frac{1}{3}}{-\frac{1}{3} + 1} = \frac{\frac{2}{3}}{+\frac{2}{3}} = +1 \quad b = -1$

$a - b = \frac{-1}{3} - (-1) = \frac{2}{3}$ ۲

$f=g \rightarrow \quad \frac{3}{2}\sin x = \sin x + \frac{\cos x}{2} \Rightarrow \frac{\sin x}{2} = \frac{\cos x}{2}$ (5

$\xrightarrow{x=[0,\pi]} \quad x = \frac{\pi}{4} \quad \Rightarrow f'(\frac{\pi}{4})$

$f'(\frac{\pi}{4}) = \cos\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4}$

$f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4} = \frac{3\sqrt{2}}{4}$

$y = \frac{\sqrt{2}}{4}x + b$

$(\frac{\pi}{4}, \frac{3\sqrt{2}}{4}) \qquad \frac{\sqrt{2}}{4}(\frac{\pi}{4}) + b = \frac{3\sqrt{2}}{4}$

$b = \frac{3\sqrt{2}}{4} - \frac{\pi\sqrt{2}}{16} = \frac{(12-\pi)\sqrt{2}}{16}$

$y = \frac{\sqrt{2}}{4}x + \frac{(12-\pi)\sqrt{2}}{16} = 0 \qquad x = \frac{\frac{(\pi-12)\cancel{\sqrt{2}}}{\cancel{16}\,4}}{\frac{\cancel{\sqrt{2}}}{\cancel{4}}} = \frac{\pi-12}{4}$

۲

$f' = 6x^2 - 6x - 12 = 6(x-2)(x+1)$ (6

$A \rightarrow x=2 \quad y=-19 \qquad B \rightarrow x=-1 \quad y=8$

$m_{AB} = \frac{8-(-19)}{-3} = -9$

۲

$f' = -9 \rightarrow 6x^2 - 6x - 12 = -9 \qquad 6x^2 - 6x - 3 = 0$

$\Delta > 0 \Rightarrow$ ۲ نقطه

$f' = 3kx^2 + (2k+2)x \qquad f'' = 6kx + 2k + 2 = 0$ (7

$x = \frac{-2k-2}{6k} < 0 \qquad \begin{array}{ccccc} & -1 & & 0 & \\ - & | & + & | & - \end{array} \qquad \xrightarrow{k<0} k = (-\infty, -1)$

۱/۵

$y = x^2(kx+k+1) = \left(\frac{4k^2+8k+4}{36k^2}\right)\left(\frac{-2k-2}{6} + k + 1\right)$

$= \left(\frac{k+2}{9}\right)\left(\frac{4k+4}{6}\right) = \frac{2k^2+6k+4}{27} > 0 \qquad 2(k+2)(k+1) > 0$

$\begin{array}{ccccc} & -2 & & -1 & \\ + & | & - & | & + \end{array} \qquad k < -2 \qquad k = (-\infty, -2)$

(۸

$m = f' = 3x^2 + 2ax + b$

$f(-1) = -4 \rightarrow -1 + a - b - 1 = -4$ ۱/۵ $a - b = -2$

$$m = \frac{(x^3 + ax^2 + bx - 1) + 4}{x+1} = 3x^2 + 2ax + b$$

$x^3 + ax^2 + bx + 3 = 3x^3 + (2a+3)x^2 + (b+2a)x + b$

(۹ $f' = 3x^2 + 2ax + b$

$f(0) = 4 \rightarrow c = 4$

$f'(0) = 0 \rightarrow b = 0$ $f' = 0 \rightarrow x = 0 , \frac{-2a}{3}$

$$f\left(-\frac{2a}{3}\right) = \frac{-8a^3}{27} + \frac{4a^3}{9} + 4 = 0$$

$$\frac{-8a^3 + 12a^3}{27} = -4 \qquad \frac{a^3}{27} = -1 \qquad a = -3$$

$x = -\frac{2a}{3} = \frac{-2(-3)}{3} = 2$ ← طول نقطه min

(۱۰ $f' = 4x^3 - 12x = 4x(x^2 - 3)$

$-\sqrt{3}$		0		$\sqrt{3}$	
$-$		$+$		$-$	$+$
min		max		min	

$A \rightarrow (-\sqrt{3} , -4)$

$B \rightarrow (\sqrt{3} , -4)$

$m_{AB} = 0$ $y = -4$

$f'' = 12x^2 - 12$

	-1		$+1$	
$+$		$-$		$+$

$C \rightarrow (1 , 0)$

$D \rightarrow (-1 , 0)$

$m_{CD} = 0$ ۲ $y = 0$

این ۲ پاره خط موازی اند.

۱—

$$\lim_{x\to 0^+}\frac{f(x)}{x}=0 \rightarrow \lim_{x\to 0^+}\frac{\cos^3(2x)+ax^2+b}{x}=0 \rightarrow \lim_{x\to 0^+}\frac{1+b}{x}=0 \rightarrow \boxed{b=-1}$$

$$\lim_{x\to 0^-}\frac{f'(x)}{x}=2=\lim\frac{-6\sin(2x)\cos^2(2x)+2ax}{x}=2 \xrightarrow{\text{هم‌ارزی}}$$

$$\lim_{x\to 0^-}\frac{(-6\times 2x)+2ax}{x}=2 \rightarrow 2a-12=2 \rightarrow 2a=14 \rightarrow \boxed{a=7}$$

$$a+b=7-1=6$$

۳—

$$m=\frac{6-(-12)}{2.5-(-0.5)}=\frac{18}{3}=6 \rightarrow y=6x-9$$

$$\frac{a}{2x-1}=6x-9 \rightarrow 12x^2-24x+9-a=0 \xrightarrow{\Delta=0} \cancel{24}^2-\cancel{4}(\cancel{12})(9-a)=0 \rightarrow 12-9+a=0$$

$$\cancel{24\times 6}\quad 12$$

$$a=-3$$

$$f(5)=\frac{-3}{2(5)-1}=\frac{-3}{9}=-\frac{1}{3}$$

۷—

$$y'=3kx^2+2(k+1)x \rightarrow y''=6kx+2(k+1)=0 \rightarrow x=\frac{k+1}{-3k}$$

1

$$\frac{-(k+1)}{3k}<0 \rightarrow \begin{array}{ccc} & -1 & 0 \\ - & + & - \end{array} \rightarrow k<-1 \text{ یا } k>0$$ ← نقطه عطف در ناحیه دوم است پس

2

$$-\frac{(k+1)}{3k}(k)+(k+1)>0 \rightarrow \frac{-(k+1)}{3}+k+1>0 \rightarrow \frac{2k+2}{3}>0 \rightarrow k>-1$$

$$1\cap 2 \rightarrow k>0$$

به ازای هیچ مقدار k منفی و صحیح جواب ندارد!

۸—

$$x_{\text{عطف}}=\frac{-b}{3a}=\frac{-a}{3} \rightarrow x=-\frac{a}{3} \rightarrow \frac{-a}{3}=-1 \rightarrow a=3$$

$$f(-1)=-4 \rightarrow -1+3-b-1=-4 \rightarrow b=-5$$

$$\frac{a}{b}=\frac{3}{5}$$