سینا صفری دیز — تکلیف ۲۷ — دوازدهم ریاضی ۵

۱

$\lim_{x\to 0}\frac{f(x)}{x}=0 \xrightarrow{\frac{0}{0}} f(0)=0=1+b\Rightarrow b=-1$

$\lim_{x\to 0}\frac{f'(x)}{1}=0$ (Hop)

$\lim_{x\to 0^-}\frac{f'(x)}{x}=2 \overset{Hop}{\Rightarrow} \lim_{x\to 0^-}\frac{f''(x)}{1}=2$

$f'(x)=2\cos^2(2x)\times(-2\sin 2x)+2a x$

$f''(0)=-4\cos 2x\times 2\cos 2x+2a=2a-12=2\Rightarrow a=7$

$\Rightarrow a+b=6$

۲

$xy-1=y_1\Rightarrow x=\pm\sqrt{y_1+1}$

$y'=2x$

$\Rightarrow f'(x_1)\times f'(x_2)=-1=2x_1\times 2x_2=4\left(\sqrt{y_1+1}\right)\left(-\sqrt{y_1+1}\right)=-4(y_1+1)\Rightarrow y_1=\frac{-3}{4}\Rightarrow 4y_1=-3$

۳

$m=\frac{4-(-14)}{2a-(-5a)}=\frac{18}{7a}=2\Rightarrow y=4x-9$

$\Rightarrow \frac{a}{4x-1}=4x-9\Rightarrow 12x^2-12x+9-a=0\overset{\Delta=0}{\Rightarrow} a=6\Rightarrow f(x)=\frac{6}{4x-1}\Rightarrow f(3)=\frac{3}{3}$

۴

$\frac{1+a}{a+1}=2+b\Rightarrow b=-1$

$\frac{1-a^2}{(a+1)^2}=2\Rightarrow 1-a^2=2a^2+4a+2\Rightarrow 3a^2+4a+1=0\Rightarrow \begin{cases} a=-1 \text{ غ ق ق} \\ a=-\frac{1}{3}\end{cases}\Rightarrow a-b=\frac{2}{3}$

۵

$\sin x+\frac{1}{2}\cos x=\frac{3}{2}\sin x\Rightarrow \frac{1}{2}\cos x=\frac{1}{2}\sin x\Rightarrow \cos x=\sin x\Rightarrow x=\frac{\pi}{4}$

$\Rightarrow f'(x)=\cos x-\frac{1}{2}\sin x\Rightarrow f'\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{4}=\frac{\sqrt{2}}{4}$

$f\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{4}=\frac{3\sqrt{2}}{4}\Rightarrow y=\frac{\sqrt{2}}{4}x-\frac{\sqrt{2}\pi}{16}+\frac{3\sqrt{2}}{4}=0\Rightarrow x=3-2\pi$

۶

$f'(x)=6x^2-6x-12=0\Rightarrow \begin{cases} x=-1\Rightarrow y=8 \\ x=2\Rightarrow y=-19\end{cases}\Rightarrow m=\frac{8-(-19)}{-1-2}=-9\Rightarrow 6x^2-6x-12=-9\Rightarrow 6x^2-6x-3=0\overset{\Delta>0}{\Rightarrow}$ ۲ نقطه

۷

$y'=3kx^2+2(k+1)x\Rightarrow y''=6kx+2(k+1)=0\Rightarrow x=\frac{-2(k+1)}{6k}=\frac{-(k+1)}{3k}\Rightarrow \frac{-(k+1)}{3k}<0$

	-1	0	
	−	+	−

$k<-1$ (I)

$y=kx^3+(k+1)x^2=x^2(kx+k+1)\Rightarrow\left(\frac{-(k+1)}{3k}\right)^2\left(\frac{2k+2}{3}\right)>0\Rightarrow\frac{2(k+1)}{3}>0\Rightarrow k>-1$ (II)

$(I)\cap(II)=\emptyset$ — غیر ممکن

۸

$y=x^3+ax^2+bx-1\Rightarrow y'=3x^2+2ax+b\Rightarrow y''=6x+2a\Rightarrow x=\frac{-2a}{6}=-1\Rightarrow a=3$

$-1+a-b-1=-4\Rightarrow a-b=-2\Rightarrow b=5$

حاصل از نمی تبدیل نقطه عطف $\frac{a}{b}=\frac{3}{5}$

۹

$c=4$ $(0,4)$ ماکزیمم نسبی $\Rightarrow f'(0)=0\Rightarrow f'(x)=3x^2+2ax+b\Rightarrow b=0\Rightarrow f'(x)=3x^2+2ax=x(3x+2a)\Rightarrow x_1=-\frac{2a}{3}$

$\Rightarrow y_1=\frac{-8a^3}{27}+\frac{4a^2}{9}+4=0\Rightarrow\frac{4a^2}{27}=-4\Rightarrow a^2=-27\Rightarrow a=-3\Rightarrow x_1=2$

۱۰

$f(x)=x^4-8x^2+6\Rightarrow f'(x)=4x^3-16x=4(x^2-4)$

$f'(x)=12x^2-12\Rightarrow \begin{cases} x_C=1\Rightarrow y_C=0 \\ x_D=-1\Rightarrow y_D=0\end{cases}\Rightarrow d_{CD}: y=0\Rightarrow d_{CD}\parallel d_{AB}\Rightarrow \alpha=0^\circ$

	$-\sqrt{4}$	0	$\sqrt{4}$	
y'	−	+	−	+

$\Rightarrow \begin{cases} x_A=\sqrt{4}\Rightarrow y_A=-6 \\ x_B=-\sqrt{4}\Rightarrow y_B=-6\end{cases}\Rightarrow d_{AB}: y=-6$