نام: دنیا احمدی   شماره کلاس: ۲۷   مقطع: دوازدهم دختران B

(۱)

$$\lim \frac{f(x)}{x} = 0 \to \lim \frac{\cos^2(2x) + ax^2 + b}{x} = \frac{ax^2 + b + 1}{x} \xrightarrow{ax=0} b + 1 = 0 \to b = -1$$

$$\lim \frac{f'(x)}{x} = 2 \to \frac{2\cos^2(2x) \times 2x - \sin(2x) + 2ax}{x} = -4\cos^2(2x) \cdot \frac{\sin 2x}{x} + \frac{2ax}{x}$$

$$= -4\cos^2(2x) \times 2 + 2a = -12 + 2a = 2 \to a = 7 \to a + b = 7 - 1 = 6$$

(۲)

$$\xrightarrow{\text{شیب}} f'(x) = 2x \xrightarrow{\text{عمود}} m = \frac{-1}{m} \to 2x = \frac{1}{2x} \to 4x^2 = 1 \to x = \pm\frac{1}{2} \to y = (\pm\tfrac{1}{2})^2 - 1 = -\frac{3}{4}$$

$$\to y_{x_1} + y_{x_2} = 2(-\tfrac{3}{4}) = -\tfrac{3}{2}$$

(۳)

$$f(x) = \frac{a}{2x-1} \to f'(x) = \frac{-2a}{(2x-1)^2} \to y = ax + b \to y = \frac{-2a}{(2x-1)^2} + b \xrightarrow{(3,6)} 6 = \frac{-5a}{16} + b \Rightarrow b = 6 + \frac{5a}{16}$$

$$(-1,-12) \to -12 = \frac{a}{4} + b \Rightarrow b = \frac{-a}{4} - 12$$

$$\to 6 + \frac{5a}{16} = -12 - \frac{a}{4} \to a = -32$$

$$f(x) = \frac{-32}{2x-1} \Rightarrow f(5) = \frac{-32}{9}$$

(۴)

$$y = \frac{x+a}{ax+1} \to y' = \frac{1-a^2}{(ax+1)^2} \xrightarrow[\text{شیب}]{\text{برابر}} y'_1 = y'_2 \to 2 = \frac{1-a^2}{(ax+1)^2} \xrightarrow{x=1} 3a^2 + 4a + 1 = 0 \to a = -1 \times$$ (غیرقابل قبول) $\quad a = \frac{-1}{3}$

$$\xrightarrow{\text{عرض برابر}} y_1 = y_2 \to y = \frac{x+a}{ax+1} \overset{x=1}{=} \frac{1+a}{1+a} = 1 = 2 + b \to b = -1 \to a - b = \frac{2}{3}$$

(۵)

$$g(x) = f(x) \Rightarrow \frac{3}{2}\sin(x) = \sin x + \frac{1}{2}\cos x \to \sin x = \cos x \xrightarrow{[0,\pi]} x = \frac{\pi}{4}$$

$$f'(x) = \cos x - \frac{1}{2}\sin(x) \xrightarrow{x=\frac{\pi}{4}} f'(x) = \frac{\sqrt{2}}{4} \to A\left|\begin{matrix}\frac{\pi}{4} \\ \frac{3\sqrt{2}}{4}\end{matrix}\right. \to \frac{3\sqrt{2}}{4} = \frac{\sqrt{2}}{4} \times \frac{\pi}{4} + b \to b = \frac{(3-\pi)\sqrt{2}}{4}$$

$$y = \frac{\sqrt{2}}{4}x + \frac{(3-\pi)\sqrt{2}}{4} \xrightarrow{x=0} y = \frac{(3-\pi)\sqrt{2}}{4}$$

(۶)

$$f(x) = 2x^3 - 3x^2 - 12x + 1 \to f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$$

$$\xrightarrow[f'(x)=0]{\text{اکسترمم}} \begin{matrix} x = 2 \to A\left|\begin{matrix}2 \\ -19\end{matrix}\right. \\ x = -1 \to B\left|\begin{matrix}-1 \\ 8\end{matrix}\right. \end{matrix} \to m_{AB} = \frac{-19}{3} \to f'(x) = \frac{-19}{3} \to 6(x^2 - x - 2) = \frac{-19}{3} \to x^2 - x - \frac{17}{18} = 0$$

$$\xrightarrow{\Delta > 0} \text{دو نقطه}$$

(۷)

$$y = kx^3 + (k+1)x^2 \to y' = 3kx^2 + (2k+2)x \to y'' = 6kx + (2k+2) \xrightarrow[y''=0]{\text{عطف}} 6kx + 2k + 2 = 0$$

$$\to 3kx = -k - 1 \to x = \frac{-k-1}{3k} \xrightarrow[x>0]{\text{طول مثبت}} \frac{-k-1}{3k} > 0 \xrightarrow{k<0} k \in (-\infty, -1) \quad (1)$$

$$\xrightarrow[\text{عرض مثبت}]{y>0} x^2(kx + k + 1) > 0 \to kx + k + 1 > 0 \to k\left(\frac{-k-1}{3k}\right) + k + 1 > 0 \to k + 1 > 0 \to k > -1 \to k \in (-1, +\infty) \quad (2)$$

صرف نظر

$$(1) \cap (2) \to \{\varnothing\} \to k \text{ هیچ مقدار}$$

⑧

$f(x) = x^3 + ax^2 + bx - 1$ $\qquad$ $f'(x) = 3x^2 + 2ax + b$

$f(-1) = -4 \rightarrow -4 = -1 + a - b - 1 \rightarrow a - b = -2$ $\qquad$ $f'(-1) = 3 - 2a + b$

$\rightarrow y = ax + b \longrightarrow -4 = -(3 - 2a + b) + b \rightarrow$

⑨

$(0,4) \xrightarrow{f(x)} c = 4$ $\qquad$ اکسترمم $\xrightarrow[x=0]{f'(x)=0} b = 0$ $\qquad$ $f''(x) = 6x + 2a \xrightarrow[f''(x_0)=0]{\text{عطف}} x_0 = \frac{-a}{3}$

$y_0 = \frac{y_{min} + y_{max}}{2} = \frac{4+0}{2} = 2 \xrightarrow[f(x_0)]{(\frac{-a}{3}, 2)} 2 = \frac{-a^3}{27} + a(\frac{a^2}{9}) + 0 + 4 \Rightarrow \frac{2a^3}{27} = -2 \rightarrow a = -3$

$f'(x) = 3x^2 - 6x = 3x(x-2)$ $\begin{cases} x=0 \rightarrow \text{ماکزیمم} \\ x=2 \end{cases}$

⑩

$f'(x) = 4x^3 - 6x \rightarrow 2x(2x^2 - 3) = 0 \longrightarrow x = 0$ $\qquad$ $x = \pm\sqrt{\frac{3}{2}}$

x	$-\sqrt{\frac{3}{2}}$	0	$+\sqrt{\frac{3}{2}}$
y'	$-$	$+$	$-$ $+$
y	↘	↗	↘ ↗

A $\left|\begin{matrix} -\sqrt{\frac{3}{2}} \\ -\frac{9}{4} \end{matrix}\right.$ $\qquad$ B $\left|\begin{matrix} \sqrt{\frac{3}{2}} \\ -\frac{9}{4} \end{matrix}\right.$ $\rightarrow m_{AB} = 0$

$f''(x) = 12x^2 - 6 \rightarrow 6(2x^2 - 1) = 0 \rightarrow x = \pm\frac{1}{\sqrt{2}}$ $\qquad$ D $\left|\begin{matrix} \frac{1}{\sqrt{2}} \\ -\frac{9}{4} \end{matrix}\right.$ $\qquad$ C $\left|\begin{matrix} -\frac{1}{\sqrt{2}} \\ -\frac{9}{4} \end{matrix}\right.$ $\rightarrow m_{DC} = 0 \rightarrow$ [illegible]