سارینا عابدینی

۹۵/۱۰/۲۶ آفرین

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$$f'(x)=-6\sin 2x\cos^2 2x+2ax \xrightarrow{\lim_{x\to 0^-}} \frac{-6\sin 2x\cos^2 2x+2ax}{x}$$

$$\xrightarrow{\text{hop}} \lim_{x\to 0^-}\frac{-2(12\sin 2x\cos^2 2x+6\sin 2x\cdot 2(\cos 2x)(-2\sin 2x)+2a)}{1}=2$$

$$\hookrightarrow (-6)(2)+2a=2 \quad a=7$$

$$\lim_{x\to 0^+}\frac{\cos 2x}{x}=\lim_{x\to 0^+}\frac{\cos^2 2x+ax+b}{x}=0 \to \cos^2 0+b=0 \to b=-1$$

$$a+b=6$$

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۲-

$$y=x^2-1 \to y'=2x \to 2x(-2x)=-1 \to 4x^2=1 \begin{cases} x=\frac{1}{2} \\ x=-\frac{1}{2} \end{cases}$$

$$x=\pm\frac{1}{2}$$

$$y=\frac{1}{4}-1=-\frac{3}{4} \to -\frac{3}{4}-\frac{2}{4}=-\frac{5}{4}$$

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۳-

$$m=\frac{8+12}{10+10}=2 \to y=8x-9$$

$$f(x)=\frac{9}{2x-1}=8x-9 \to a=(8x-9)(2x-1)$$

$$f'(x)=\frac{-2a}{(2x-1)^2}=8 \to a=-2(4x^2-4x+1)$$

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$$-16x^2+16x-3=8x-9 \to x=1 \text{ و } a=-2 \to f(x)=\frac{-4}{9}=-\frac{1}{2}$$

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۴-

$y = 2x + b \xrightarrow{\text{مشتق}} y' = 2$ ، $x=1$

$y = \frac{x+a}{ax+1} \xrightarrow{\text{مشتق}} y' = \frac{(ax+1) - a(x+a)}{(ax+1)^2} = \frac{1-a^2}{(1+ax)^2}$ (۲)

$2 = \frac{1-a}{1+a} \longrightarrow a = -\frac{1}{2}$ ، $y = \frac{x-\frac{1}{2}}{1-\frac{x}{2}} \xrightarrow{x=1} y=1 \rightarrow 2+b=1 \rightarrow b=-1$

۵-

$g(x) = f(x) \rightarrow \frac{2}{2}\sin x = \sin x + \frac{1}{2}\cos x \rightarrow \sin x = \cos x \rightarrow x = \frac{\pi}{4}$

$f'(x) = \cos x - \frac{1}{2}\sin x \longrightarrow f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{4}$ ، $f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4} = \frac{3\sqrt{2}}{4}$ (۱/۵)

$\frac{\sqrt{2}}{4}x - \frac{\sqrt{2}\pi}{8} + \frac{3\sqrt{2}}{4} = 0 \rightarrow x = \frac{\pi}{4}$

۶-

$f'(x) = 6x^2 - 4x - 12 = 0$ ، $x=1 \rightarrow (-6, 1)$ ، $x=2 \rightarrow (-8, 2)$ $\rightarrow m_{AB} = \frac{-2}{2} = -1$

$f'(x) = 2 \rightarrow 6x^2 - 4x - 2 = 0 \rightarrow \Delta = 16 + 24 = 40 > 0$ ← دو نقطه (۲)

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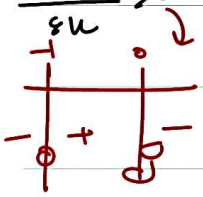
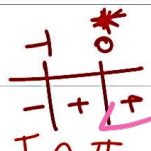
$f'(x)=4x^2-4x-12=0 \begin{cases} x=2 \\ x=-1 \end{cases}$ ، $f''(x)=6kx+2k+2>0 \rightarrow x>-\frac{2k+2}{6k}$

$\frac{2k+2}{6k}>0$



$\rightarrow k\in(-\infty,-1)$ ①

$(\frac{-2k-2}{6k})(\frac{6+6k}{6})>0 \rightarrow$



$\rightarrow k\in(-1,0)$ ②

$I\cap II=\emptyset$

۸-

$(-1)^3+a(-1)^2-b-1=4 \rightarrow a-b=-2$

$x=\frac{-a}{3} \rightarrow -1=\frac{-a}{3} \rightarrow a=3 \rightarrow b=5 \rightarrow \frac{a}{b}=\frac{3}{5}$ ②

۹-

$f'(x)=3x^2+2ax+b=0 \xrightarrow{x=0} b=0$

$f'(x)=3x^2+2ax=0 \rightarrow x=\frac{-2a}{3} \rightarrow f(\frac{-2a}{3})=0$

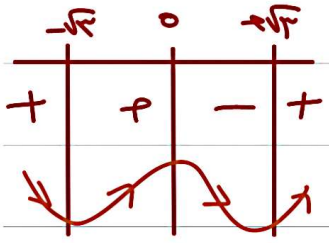
$\rightarrow (\frac{-2a}{3})^3+a(\frac{-2a}{3})^2+c=0 \rightarrow a=-3 \rightarrow \frac{-2a}{3}=1$ ②

$c=4$

۱۰-

$f'(x)=4x^3-12x \rightarrow f'(x)=0 \rightarrow x\in\{-\sqrt{3},0,\sqrt{3}\}$

$f''(x)=12x^2-12 \rightarrow f''(x)=0 \rightarrow x\in\{\pm 1\}$



$A(\sqrt{3},-6), B(-\sqrt{3},4) \rightarrow m_{AB}=0$

$C(1,0), D(-1,0) \rightarrow m_{CD}=0 \rightarrow \theta=0$ ②

ب -

$f(x) = g(x) \rightarrow \sin x + \frac{1}{2}\cos x = \frac{\sqrt{2}}{2}\sin x \rightarrow \sin x = \cos x \xrightarrow{0 < x < \frac{\pi}{2}} x = \frac{\pi}{4}$

$f(\frac{\pi}{2}) = \sin\frac{\pi}{2} + \frac{1}{2}\cos\frac{\pi}{2} = \frac{2\sqrt{2}}{2} + \frac{1}{2} = \frac{2\sqrt{2}+1}{2}$

$f'(x) = \cos x - \frac{1}{2}\sin x \rightarrow f'(\frac{\pi}{2}) = \frac{\sqrt{2}}{2} - \frac{1}{2} = \frac{\sqrt{2}-1}{2}$

معادله خط به شیب $\frac{\sqrt{2}-1}{2}$ → $y - \frac{2\sqrt{2}+1}{2} = \frac{\sqrt{2}-1}{2}(x - \frac{\pi}{2})$ $y=0$ $\rightsquigarrow$ $\frac{\sqrt{2}-1}{2}(\frac{\pi}{2} - x) = \frac{2\sqrt{2}+1}{2}$ → $x = \frac{\pi}{4} - 3$