سوال ①

سوال ②

سوال ③

$f(x)=\frac{a}{2x-1}$ $\quad m_{\text{خط کمکی}}=\frac{-12-4}{-0/5-2/5}=\frac{-18}{-3}=4$

$(2/5,4)$ $(-0/5,-12)$ معادله خط کمکی $y-4=4(x-2/5)\Rightarrow y=4x-9$

$f(a)=?$ $\quad \frac{a}{2x-1}=4x-9\Rightarrow a=(4x-9)(2x-1)\Rightarrow a=12x^2-26x+9$

$12x^2-18x-4x+9$

$12x^2-34x+9-a=0\Rightarrow \Delta=0\Rightarrow a=-c\Rightarrow f(x)=\frac{-c}{2x-1}\Rightarrow f(x)=f(a)$

$\boxed{f(a)=-\frac{1}{3}}$

سوال ④

$y=2x+b\Rightarrow y'=2$

در $x=1$

$y=\frac{x+a}{ax+1}\Rightarrow y'=\frac{(ax+1)\times 1-a(x+a)}{(ax+1)^2}=\frac{1-a^2}{(ax+1)^2}$

$y'(1)=\frac{1-a^2}{(a+1)^2}=2\Rightarrow 2(a+1)^2=(1-a)^2\Rightarrow 2(a^2+1+2a)=(1-a)^2\Rightarrow$

$2a^2+4a+1=0\Rightarrow (a+1)(2a+1)=0$ $\begin{cases} a=-1\Rightarrow y=-1 \Rightarrow \text{ثابت} \times \\ \boxed{a=-1/2}\checkmark \end{cases}$

$a=-1/2\Rightarrow y=\frac{x-1/2}{1/2x+1}=\frac{2x-1}{-x+2}\xrightarrow{x=1}y=1$

$A(1,1)\Rightarrow y=2x+b\Rightarrow 1=2+b\Rightarrow \boxed{b=-1}$

$\boxed{a-b=-\frac{1}{2}+1=2/5}$

سؤال ۵

$\sin x + \frac{1}{2}\cos x = \frac{\sqrt{2}}{2}\sin x \Rightarrow \sin x = \cos x \Rightarrow x = \frac{\pi}{4}$

$A\left(\frac{\pi}{4}, \frac{2\sqrt{2}}{2}\right) \Rightarrow f(x) = \cos x - \frac{1}{2}\sin x \rightarrow \boxed{f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{4}}$

$y - \frac{2\sqrt{2}}{2} = \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right) \rightsquigarrow y = 0 \Rightarrow x = \frac{\pi}{4} - 4$

سؤال ۶

$f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12 = 0 \rightarrow x^2 - x - 2 = 0 \Rightarrow \begin{cases} x_1 = -1 \Rightarrow y_1 = 8 \\ x_2 = 2 \Rightarrow y_2 = -19 \end{cases}$

$A(-1, 8)$, $B(2, -19)$ $\Rightarrow \overrightarrow{AB} = \frac{8 + 19}{-1 - 2} = -9$

$\Rightarrow 6x^2 - 6x + 12 = -9 \Rightarrow 6x^2 - 6x - 3 = 0 \rightsquigarrow \Delta > 0 \rightsquigarrow$ دو جواب دارد

۲ نقطه

سؤال ۷

$y = \frac{k}{2}x^3 - (k+2)x^2$

$y' = \frac{3}{2}kx^2 - 2(k+2)x$

$y'' = 3kx - 2(k+2) \Rightarrow y'' = 0 \Rightarrow x = \frac{2(k+2)}{3k}$

$f\left(\frac{2(k+2)}{3k}\right) = \frac{k}{2}\left(\frac{2}{3}\right)^3 \times \frac{(k+2)^3}{k^3} - (k+2)\left(\frac{2}{3}\right)^2 \times \frac{(k+2)^2}{k^2} = -\frac{8(k+2)^3}{27k^3}$

نقطه عطف $\left(\frac{2(k+2)}{3k}, -\frac{8(k+2)^3}{27k^3}\right)$ $\xrightarrow{\text{در ناحیه سوم}}$ $x_A < 0$, $y_A < 0$

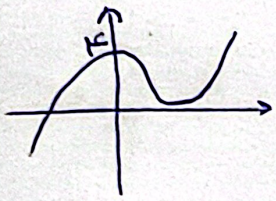
$\frac{2(k+2)}{3k} < 0 \rightarrow -2 < k < 0$ ① $-\frac{8}{27}\times\frac{(k+2)^3}{k^3} < 0 \rightarrow k + 2 > 0 \rightarrow k > -2$ ②

① ∩ ② $k \in (-2, 0)$

سؤال ۸

سؤال ۹

$f(x)=x^3+ax^2+bx+c$ نمودار $(0,4)$ ماکزیمم نسبی



$\Rightarrow f(0)=4 \Rightarrow c=4 \Rightarrow f(x)=x^3+ax^2+bx+4$

$\Rightarrow f'(x)=3x^2+2ax+b$

$\Rightarrow f'(0)=0 \Rightarrow b=0$

$\Rightarrow f(x)=x^3+ax^2+4$

$\Rightarrow f'(x)=3x^2+2ax=0 \Rightarrow \begin{cases} x=0 \\ 3x+2a=0 \Rightarrow x=\frac{-2a}{3} \end{cases}$ طول می‌نیمم

$y=0$

$f(\frac{-2a}{3})=0 \Rightarrow (\frac{-2a}{3})^3+a(\frac{-2a}{3})^2+4=0 \Rightarrow \frac{4a^3}{27}=-4 \Rightarrow a^3=-27$

$\Rightarrow a=-3 \Rightarrow x=\frac{-2a}{3}=\frac{-2(-3)}{3}=2$ می‌نیمم $(2,0)$

سؤال ۱۰

$f(x)=x^4-4x^2+2 \Rightarrow f'(x)=4x^3-8x=4x(x^2-2)$

$x: \quad 0 \quad -\sqrt{2} \quad \sqrt{2}$

$f': \quad 0 \quad + \quad 0 \quad - \quad 0 \quad + \quad$ (min, min)

$A(-\sqrt{2},-2), B(\sqrt{2},-2)$

$f''(x)=12x^2-8=4(3x^2-2)$

$3x^2-2=0 \Rightarrow x=\pm\sqrt{\frac{2}{3}}$

$x: \quad -\sqrt{\frac{2}{3}} \quad \sqrt{\frac{2}{3}}$

$f'': \quad + \quad 0 \quad - \quad 0 \quad +$ (∪, ∩, ∪)

f: ↑ ↓ ↑

ب) $m_{AB}=\frac{-2+2}{\sqrt{2}+\sqrt{2}}=0$

ج) $m_{OA}=\frac{0-0}{1+1}=0$

$\Rightarrow$ شیب‌ها برابرند $\Rightarrow$ موازی‌اند $\Rightarrow$ زاویه $=0$