

$$y = \frac{n^2 n + 2}{n-1} \Rightarrow y n = y - n^2 + n + 2$$

$$\Rightarrow n^2 + n - y n - 2 + y = 0$$

$$\Rightarrow n^2 + (1-y)n - 2 + y = 0 \quad \Delta = (1-y)^2 - 4(-2+y) = (1-y)^2 + 8 - 4y$$

$$\Delta \Rightarrow (1-y)^2 - 4(2-y) \Rightarrow 1 - 2y + y^2 - 8 + 4y = y^2 + 2y - 7$$

$$y^2 + 2y - 7 = 0 \Rightarrow y = \frac{-2 \pm \sqrt{4 + 28}}{2} = \frac{-2 \pm \sqrt{32}}{2} = \frac{-2 \pm 4\sqrt{2}}{2} = -1 \pm 2\sqrt{2}$$

$$y = -1 + 2\sqrt{2}$$

$$R_f = (-\infty, 1] \cup [2\sqrt{2} - 1, +\infty)$$

الف) $y = \frac{n^2}{n+1}$ $\Delta = \frac{b^2 - 4ac}{4a^2} = \frac{1 - 4}{4} = -\frac{3}{4} < 0$ $\Rightarrow R_f = \emptyset$

$$\frac{2(28)}{14} \pm \frac{1}{2} \Rightarrow R_f = \left[-\frac{1}{2}, +\infty \right)$$

ب) $y = \sqrt{n^2 + 4n + 5}$ $\Delta = \frac{b^2 - 4ac}{4a^2} = \frac{16 - 20}{4} = -1 < 0$ $\Rightarrow R_f = \emptyset$

$$\Rightarrow \Delta \Rightarrow \sqrt{16 - 20} = \sqrt{-4} = 2i \Rightarrow R_f = [2, +\infty)$$

ج) $\sqrt{n^2 - 3n + 2} \Rightarrow \sqrt{1R} \Rightarrow R_f = [0, +\infty)$

د) $\log(n^2 - 4n^2 + 4n + 2) = \log 1R = R_f(-\infty, +\infty)$

الف) $y = \frac{n+1}{n-4} \Rightarrow R_f = 1R - \left\{ \frac{1}{4} \right\} \Rightarrow R_f = 1R - \left\{ \frac{1}{4} \right\}$

ب) $y = \sqrt{\frac{2n+1}{n-2}} = \sqrt{1R - \{2\}} = R_f = [0, +\infty) - \{2\}$

$$ج) y = \frac{x^2 + \varepsilon}{\sqrt{x^2 + 3}} \Rightarrow \left(\sqrt{x^2 + 3} \right)^2 = 1 - \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}}$$

$$\text{نقطة (نقطة) } x=0 \Rightarrow \sqrt{0+3} + \frac{1}{\sqrt{0+3}} = \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{\varepsilon\sqrt{3}}{3}$$

$$R_f = \left[\frac{\varepsilon\sqrt{3}}{3}, +\infty \right)$$

$$د) y = x^2 + \frac{1}{x^2 + \varepsilon} \Rightarrow x^2 + \varepsilon + \frac{1}{x^2 + \varepsilon} - \varepsilon$$

$$\Rightarrow x=0 \Rightarrow y = \varepsilon + \frac{1}{\varepsilon} - \varepsilon = \frac{1}{\varepsilon} \Rightarrow R_f = \left[\frac{1}{\varepsilon}, +\infty \right)$$

$$سؤال ٤) ٢) = y = \frac{3x^2 + \varepsilon}{x^2 - 1} \Rightarrow yx^2 - y = 3x^2 + \varepsilon$$

$$3x^2 - yx^2 + \varepsilon + y = 0 \Rightarrow x = \frac{0 \pm \sqrt{0 - 4(3-y)(\varepsilon+y)}}{2(3-y)} \Rightarrow$$

$$(3-y)x^2 + \varepsilon + y$$

$$\Delta_f \Rightarrow -\varepsilon(3-y)(\varepsilon+y) \geq 0 \Rightarrow \begin{cases} \varepsilon y^2 + \varepsilon y - \varepsilon \Delta_f \geq 0 \\ y^2 + y - \varepsilon \Delta_f \geq 0 \end{cases}$$

$$\begin{array}{c} + \quad | \quad - \quad | \quad + \\ \hline \end{array}$$

①

$$2(3-y) \neq 0 \Rightarrow 3-y \neq 0 \Rightarrow y \neq 3$$

في النهاية نلاحظ أن 3 هو الحل الوحيد الذي يحققه y عند $x=0$

$$R_f = (-\infty, -\varepsilon] \cup (3, +\infty)$$

$$نذكر (٢) \Rightarrow \begin{cases} x^2 \rightarrow +\infty \Rightarrow y = -\varepsilon \\ x^2 = 0 \Rightarrow y = 3 \end{cases}$$

$$[-\infty, -\varepsilon] \cup (3, +\infty)$$

مخرج منفرد

Q1) $y = \sin x - 1 \Rightarrow y \sin x + 1 = \sin x - 1$

$1 + y = \sin x - y \sin x \Rightarrow 1 + y = \sin x (1 - y) \Rightarrow \sin x = \frac{1+y}{1-y}$

$\sin x = 1 = \frac{1+y}{1-y} \Rightarrow y = -\frac{1}{2} \rightarrow \min$

$\sin x = -1 = \frac{1+y}{1-y} \Rightarrow y = \frac{1}{2} \rightarrow \max$ $R_f = [-\frac{1}{2}, \frac{1}{2}]$

$\frac{y \sin x - 1}{\sin x + y} = \frac{-1}{1} \Rightarrow \frac{y-1}{1+y} = -1 \Rightarrow \frac{y-1}{1+y} = -1$

$R_f = [-\frac{1}{2}, \frac{1}{2}]$

$f(x) = \frac{x^2 - x + 1}{x^2 - x + 1}$ $D_f = \mathbb{R}$ $\{ \sin x \}$

$D_f = \mathbb{R}$

$f(x) = \frac{(x - \frac{1}{2})^2}{(x - \frac{1}{2})^2 + \frac{3}{4}} \Rightarrow R_f = [0, \frac{4}{7}]$

$D_f = \mathbb{R}$

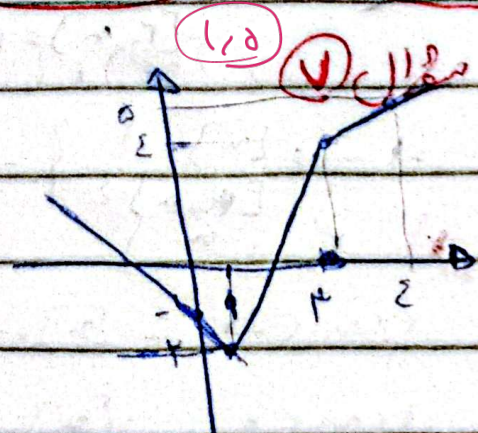
$R_f = [0, \frac{4}{7}]$

$\lim_{x \rightarrow \infty} \frac{(x - \frac{1}{2})^2}{(x - \frac{1}{2})^2 + \frac{3}{4}} = 1$

$f(x) = |x - 1| - \ln(x)$

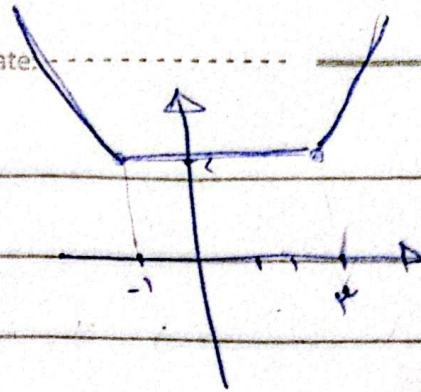
$\frac{-x + 1 - \ln(x)}{x} \quad \frac{x - 1 - \ln(x)}{x} \quad \frac{x - 1 - \ln(x)}{x+1}$

$R_f = [-1, +\infty)$



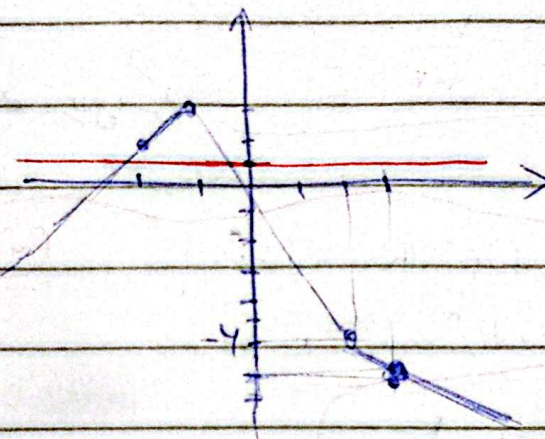
ب) $|n-2| + |n+1|$

$Rf = [-1, +\infty)$



$|n-2| = |2n+2| \leq 1$

سؤال ٢



$n=2 \rightarrow y=3$

$n=-1 \rightarrow y=2$

$n=2 \rightarrow y=3$

$n=2 \rightarrow y=3$

$\rightarrow (-\infty, 3] \cup [-\frac{1}{2}, +\infty)$

ان $[n] + [2-n] = [n] + [-n] + 2$

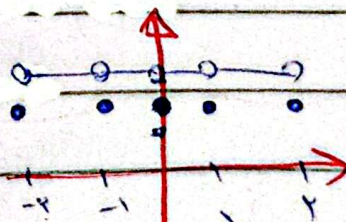
سؤال ٣

١/٨

$n \in \mathbb{Z} \Rightarrow [n] + [-n] + 2 = 2$

$n \notin \mathbb{Z} \Rightarrow [n] + [-n] + 2 = 1$

$Rf = [1, 2]$



ج) $[n]$

$-1 < n < 0 \rightarrow [n] = -1 \rightarrow Rf = [-1, 0]$

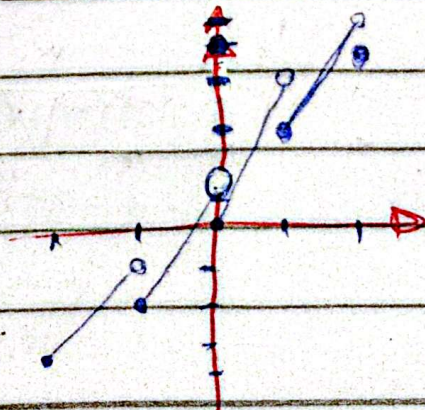
$0 < n < 1 \rightarrow [n] = 0 \rightarrow Rf = [0, 1]$

$1 < n < 2 \rightarrow [n] = 1 \rightarrow Rf = [1, 2]$

$2 < n < 3 \rightarrow [n] = 2 \rightarrow Rf = [2, 3]$

$3 < n < 4 \rightarrow [n] = 3 \rightarrow Rf = [3, 4]$

$Rf = [-1, \infty)$



NOTEBOOK
SABER

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الف) $y = \sin 5x - \sin x + 1 \xrightarrow{-1} \mu$

$\xrightarrow{1} 1$

$\xrightarrow{\frac{1}{2}} \frac{1}{2} - \frac{1}{2} + 1 = \frac{\mu}{2}$

$\textcircled{2} \textcircled{1.12}$

$R_F = \left[\frac{\mu}{2}, \mu \right]$



ب) $y = \sin^2 x + \sin^2 x + 1 \xrightarrow{1} \mu$

$\xrightarrow{0} 1$

$\xrightarrow{-1} \alpha$

$R_F = [1, \mu]$

