

$$y = \frac{x^2 + x + 2}{x - 1} \rightarrow y = \frac{x^2 + x - 2 + 4}{x - 1} \Rightarrow y = \frac{(x-1)(x+2) + 4}{x-1} = \frac{x+2}{x-1} + \frac{4}{x-1} \Rightarrow$$

$$yx - y - 4 = \frac{x+2}{x-1} \xrightarrow{x \neq 1} yx - y - 4 = x + 2 \rightarrow (y-1)x = y + 6 \Rightarrow$$

$$x = \frac{y+6}{y-1} \Rightarrow R - \{1\} = R_f$$

$\Delta \geq 0 \rightarrow y^2 - 4y - 7 \geq 0$ $y \geq 7$
 $y \leq -1$

الف) $y = \frac{2x^2 - 5x + 1}{x}$ $\Rightarrow R_f = \left[\frac{-\Delta}{4a}, +\infty \right)$
 $\Delta = 25 - 4 = 21 \Rightarrow R_f = \left[\frac{-17}{8}, +\infty \right)$

ب) $y = \sqrt{-x^2 + 4x - 8}$ $R_f = \left[0, \frac{\Delta}{4a} \right]$
 $\Delta = 16 \Rightarrow R_f = [0, 2]$

ج) $y = \sqrt{x^2 - 2x^2 + 4x + 1}$ $\Rightarrow R_y = [0, +\infty)$
 $R = \mathbb{R} \Rightarrow$

د) $\log(x^2 - 4x^2 + 7x + 4)$
 $R = (-\infty, +\infty) \subset \mathbb{R}$

ه) $y = \frac{3x+1}{2x-4} \Rightarrow x = -\frac{4y-1}{2y-3} \Rightarrow x = \frac{4y+1}{2y-3}$
 $\Rightarrow R_f = \mathbb{R} - \left\{ \frac{3}{2} \right\}$

ب) $y = \sqrt{\frac{2x+1}{x-2}} \Rightarrow y^2 = \frac{2x+1}{x-2} \Rightarrow x = \frac{-y^2-1}{y^2-2}$
 $\Rightarrow R = \mathbb{R} - \left\{ \pm 2 \right\}$
 $R_y = [0, +\infty) - \{2\}$

ج) $y = \frac{x^2+4}{\sqrt{x^2+3}}$ $\xrightarrow{x=0} y = \frac{4}{\sqrt{3}} \Rightarrow R = \left[\frac{4}{\sqrt{3}}, +\infty \right)$
 $\Rightarrow R = [-2, +\infty)$ $\min y = \frac{1}{2}$
 $R_y = \left[\frac{1}{2}, +\infty \right)$

$y = \frac{3x^2+4}{x^2-1}$ $\xrightarrow{\text{درشت اول}} x^2(y-3) = 4+y \Rightarrow x^2 = \frac{4+y}{y-3} \rightarrow \sqrt{-1} \Rightarrow R_f = (-\infty, -4] \cup (3, +\infty)$

$\xrightarrow{\text{باج دوم طرف اول}} x \neq \pm 1 \xrightarrow{\text{است}} R_f = (-\infty, -4] \cup (3, +\infty)$

$y = \frac{2 \sin x - 1}{\sin x + 3}$ $\xrightarrow{\text{درشت اول}} \sin x = \frac{3y+1}{y-2} \Rightarrow \left[-\frac{1}{3}, 2 \right) \xrightarrow{\text{درشت اول}} R_f = \left[-\frac{1}{3}, 1 \right]$

$\xrightarrow{\text{درشت دوم}} \left[-\frac{1}{3}, 2 \right) \xrightarrow{\text{درشت دوم}} R_f = \left[-\frac{1}{3}, 1 \right]$

$\sin x = 1 \rightarrow y = \frac{1}{2} \rightarrow R_y = \left[-\frac{3}{2}, \frac{1}{2} \right]$

$\sin x = -1 \rightarrow y = -\frac{3}{2}$

$$f(x) = \frac{x^2 - x + \frac{1}{\epsilon}}{x^2 - x + 1} \xrightarrow{x \rightarrow \infty} \frac{(x - \frac{1}{2})^2}{x^2 - x + 1} \Rightarrow D_f = \mathbb{R}$$

$$R_f = [0, +\infty)$$

$$x = \frac{(y-1) \pm \sqrt{(y-1)^2 - 4(y - \frac{1}{2})(y-1)}}{2(y-1)} \rightarrow 0 \leq y < 1$$

← مخرج را صفر نکند!

1
6

الف) $y = |2x-2| - |x-3|$ (رسم شکل)

$\Rightarrow R_f = [-1, +\infty)$

ب) $y = |x-3| + |x+1|$

$R_f = [4, +\infty)$

2
7

$|x-2| - |2x+2| \leq 1$

$y=1 \Rightarrow 1 = x+2 \Rightarrow x = -1$
 $\Rightarrow 1 = -3x \Rightarrow x = -\frac{1}{3}$

با توجه به شکل $\Rightarrow x = -\frac{1}{3}$

$R = (-\infty, -3] \cup [-\frac{1}{3}, +\infty)$

2
8

الف) $y = [x] + [3-x] \Rightarrow y = [x] + [-x+3]$

$\Rightarrow \begin{cases} 3 & x \in \mathbb{Z} \\ 2 & x \notin \mathbb{Z} \end{cases}$

$R = \{2, 3\}$

ب) $y = 3x - [x]$

در این بازه بردار نیست!

$R_y = [-2, 2)$

$R = \mathbb{R}$

1, 8
9

الف) $y = \sin^2 x - \sin x + 1$

$\frac{1}{16} \rightarrow \frac{1}{4} - \frac{1}{4} + 1 = \frac{3}{4}$
 $\frac{1}{-1} \rightarrow 1 - 1 + 1 = 1$
 $\frac{1}{-1} \rightarrow 1 + 1 + 1 = 3$

$\Rightarrow R = [\frac{3}{4}, 3]$

ب) $y = \sin^2 x + \sin^2 x + 1$

$\frac{1}{16} \rightarrow \frac{1}{16} + \frac{1}{16} + 1 = \frac{17}{8}$
 $\frac{1}{-1} \rightarrow 1 + 1 + 1 = 3$
 $\frac{1}{-1} \rightarrow 1 + 1 + 1 = 3$

$R = [\frac{17}{8}, 3]$

$\sin^2 x \neq -1$

1, 8
10

$\min \sin^2 x \rightarrow 0 \rightarrow 1$