

$$y = \frac{x^2 + x + 2}{x - 1} \rightarrow y = \frac{x^2 + x - 2 + 4}{x - 1} \Rightarrow y = \frac{(x-1)(x+2) + 4}{x-1} = \frac{x+2}{x-1} + \frac{4}{x-1} \Rightarrow$$

$$\frac{yx - y - 4}{x - 1} = \frac{x + 2}{x - 1} \xrightarrow{x \neq 1} yx - y - 4 = x + 2 \rightarrow (y-1)x = y + 6 \Rightarrow$$

$$x = \frac{y+6}{y-1} \Rightarrow \mathbb{R} - \{1\} = R_f$$

الف) $y = \frac{2x^2 - 5x + 1}{x}$ $\Rightarrow R_f = \left[\frac{-\Delta}{4a}, +\infty \right)$
 $\Delta = 25 - 4 = 21 \Rightarrow R_f = \left[\frac{-21}{8}, +\infty \right)$

ب) $y = \sqrt{-x^2 + 4x - 8}$ $R_f = \left[0, \frac{\Delta}{4a} \right]$
 $\Delta = 16 \Rightarrow R_f = [0, 2]$

ج) $y = \sqrt{x^2 - 2x^2 + 4x + 1}$ $\rightarrow R_y = [0, +\infty)$
 $R = \mathbb{R} \Rightarrow$

د) $\log(x^2 - 4x^2 + 12x + 5)$
 $R = (-\infty, +\infty) \subset \mathbb{R}$

ه) $y = \frac{3x+1}{2x-4} \Rightarrow x = -\frac{4y-1}{2y-3} \Rightarrow x = \frac{4y+1}{2y-3}$
 $\Rightarrow R_f = \mathbb{R} - \left\{ \frac{3}{2} \right\}$

ب) $y = \sqrt{\frac{2x+1}{x-2}} \Rightarrow y^2 = \frac{2x+1}{x-2} \Rightarrow x = \frac{-y^2-1}{y^2-2}$
 $\Rightarrow R = \mathbb{R} - \left\{ \pm \sqrt{2} \right\}$

ج) $y = \frac{x^2+4}{\sqrt{x^2+3^2}} \xrightarrow{x=0} y = \frac{4}{3} \Rightarrow R = \left[\frac{4}{3}, +\infty \right)$

د) $y = x^2 + \frac{1}{x^2+4} \Rightarrow y = x^2 + \frac{1}{x^2+4} \geq \frac{1}{4}$
 $\Rightarrow R = \left[\frac{1}{4}, +\infty \right)$

$y = \frac{3x^2+4}{x^2-1} \xrightarrow{\text{درشت کردن}} x^2(y-3) = 4+y \Rightarrow x^2 = \frac{4+y}{y-3} \rightarrow \frac{-4}{y-3} \Rightarrow R_f = (-\infty, -4] \cup (3, +\infty)$

$\xrightarrow{\text{درشت کردن}} x \neq \pm 1 \xrightarrow{\text{باید دو طرف را ۳ ضرب کنیم}} R_f = (-\infty, -4] \cup (3, +\infty)$

$y = \frac{2\sin x - 1}{\sin x + 3} \xrightarrow{\text{درشت کردن}} \sin x = \frac{3y+1}{y-2} \Rightarrow \left[-\frac{1}{3}, 2 \right) \xrightarrow{|\sin x| \leq 1} R_f = \left[-\frac{1}{3}, 1 \right]$

$\xrightarrow{\text{درشت کردن}} \left(\text{خرج از ۳ را حذف می‌کنیم} \right) \Rightarrow \left[-\frac{1}{3}, 2 \right) \xrightarrow{|\sin x| \leq 1} R_f = \left[-\frac{1}{3}, 1 \right]$

$$f(x) = \frac{x^2 - x + \frac{1}{\epsilon}}{x^2 - x + 1} \xrightarrow{x \rightarrow \infty} = \frac{(x - \frac{1}{2})^2}{x^2 - x + 1} \Rightarrow D_f = \mathbb{R}$$

$$R_f = [0, +\infty)$$

