

۱۴.۴, ۴, ۳

مشتق

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$$y = \frac{x^2 + x + 1}{x - 1} = yx - y = x^2 + x + 1 \Rightarrow x^2 - x(y - 1) + 1 + y = 0 \rightarrow x = \frac{b \pm \sqrt{\Delta}}{2a}$$

$$\Delta > 0 \rightarrow (y - 1)^2 - 4(1 + y) > 0 \rightarrow y^2 - 2y + 1 - 4 - 4y > 0 \rightarrow y^2 - 6y - 3 > 0$$

$$\frac{-1}{+b} - \frac{v}{b} \rightarrow (-\infty - 1) \cup (v, +\infty) = R$$

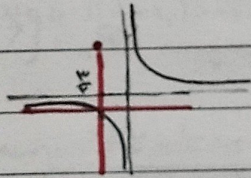
$$\text{الف) } y = 2x^2 - 2x + 1 \quad x_5 = \frac{-b}{2a} = \frac{1}{2} \Rightarrow y_5 = 2\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 1 = \frac{-1}{2} \Rightarrow R = \left[-\frac{1}{2}, +\infty\right)$$

$$\text{ب) } y = \sqrt{-x^2 + 4x - 2} \quad x_5 = \frac{-4}{-2} = 2, y_5 = 2 \Rightarrow R = (-\infty, 2] \rightarrow \sqrt{R} = [0, 2]$$

$$\text{ج) } y = \sqrt{x^2 - 4x^2 + 4x + 1} \quad R = R \rightarrow \sqrt{R} = [0, +\infty)$$

$$\text{د) } y = \lg(x^2 - 4x^2 + 4x + 1) \quad R = R \rightarrow \lg R = R \quad \left(\begin{array}{l} \text{مشتق} \\ \text{نزدیک به صفر است} \end{array} \right)$$

$$\text{هـ) } y = \frac{3x + 1}{2x - 4} \quad R = R - \left\{ \frac{3}{2} \right\}$$



$$\text{و) } y = \sqrt{\frac{4x + 1}{x - 2}} \quad R = R - \left\{ 2 \right\} \rightarrow \sqrt{R} = [0, +\infty) - \left\{ 2 \right\}$$

$$\text{ز) } y = \frac{x^2 + 4}{\sqrt{x^2 + 3}} = \frac{(\sqrt{x^2 + 3})^2 + 1}{\sqrt{x^2 + 3}} = \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}} \xrightarrow{\min} \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \Rightarrow R = \left[\frac{2\sqrt{3}}{3}, +\infty \right)$$

$$\text{ح) } y = 2x^2 + \frac{1}{2x^2 + 4} \quad 4 = 2x^2 + 4 + \frac{1}{2x^2 + 4} - 4 \Rightarrow R = \left(\frac{1}{4}, +\infty \right) = \left(\frac{1}{4}, +\infty \right)$$

$$y = \frac{3x^2 + 4}{x^2 - 1} \quad \text{اصل: } yx^2 - y = 3x^2 + 4 \Rightarrow x^2 = \frac{y + 4}{y - 3} \Rightarrow x = \pm \sqrt{\frac{y + 4}{y - 3}} \Rightarrow R_y = \left(-\infty, -4 \right] \cup \left(3, +\infty \right)$$

$$\begin{array}{l} x^2 = 0 \rightarrow y = -4 \\ x^2 = +\infty \rightarrow y = 3 \end{array} \Rightarrow (-\infty, -4] \cup (3, +\infty)$$

$$y = \frac{r \sin x - 1}{\sin x + r} \text{ جابجی: } y(\sin x) + ry = r \sin x - 1 \Rightarrow \sin x = \frac{-ry + 1}{y - r} \Rightarrow -1 \leq \frac{-ry + 1}{y - r} \leq 1$$

$$\Rightarrow \frac{-ry + 1}{y - r} \leq 1 \Rightarrow ry + 1 \leq r - y \Rightarrow ry \leq 1 \Rightarrow y \leq \frac{1}{r}$$

$$\Rightarrow \frac{-ry + 1}{y - r} \geq -1 \Rightarrow -(r - y) \leq ry - 1 \Rightarrow -r \leq ry \Rightarrow y \geq -\frac{r}{r}$$

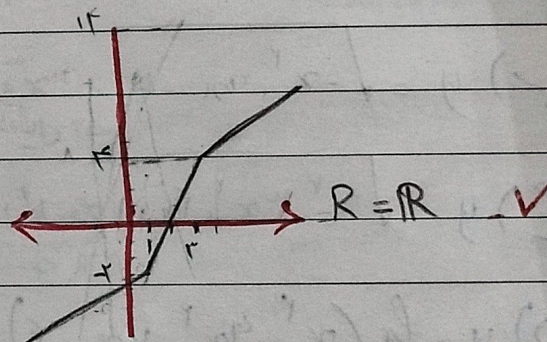
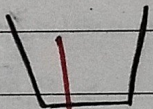
$$\left. \begin{array}{l} y \leq \frac{1}{r} \\ y \geq -\frac{r}{r} \end{array} \right\} -\frac{r}{r} \leq y \leq \frac{1}{r}$$

$$\begin{array}{l} \text{!- } \sin x = 1 \rightarrow y = \frac{r-1}{1+r} = \frac{1}{r} \text{ فتح نقاط} \\ \text{!- } \sin x = -1 \rightarrow y = \frac{-r-1}{-1+r} = -\frac{r}{r} \text{ مقصور} \end{array} \Rightarrow \left[-\frac{r}{r}, \frac{1}{r} \right]$$

$$f(x) = \frac{x^r - x + \frac{1}{r}}{x^r - x + 1} = \frac{(x - \frac{1}{r})^r}{(x - \frac{1}{r})^r + \frac{r}{r}} \quad D_f = x^r - x + 1 \neq 0 \xrightarrow{\Delta < 0} D_f = \mathbb{R}$$

$$\begin{array}{l} \text{نقطہ صفر} (x - \frac{1}{r})^r = 0 \rightarrow \frac{0}{0 + \frac{r}{r}} = 0 \\ \text{شعاع} (x - \frac{1}{r})^r \rightarrow +\infty \rightarrow \frac{+\infty}{+\infty} = 1 \end{array} \left. \vphantom{\begin{array}{l} \text{نقطہ صفر} \\ \text{شعاع} \end{array}} \right\} R = (0, 1)$$

$$\text{الف) } y = |rx - r| - |x - r| \rightarrow \begin{array}{l} x < -1 \quad -1 \leq x \leq r \quad x > r \\ -x - 2r + r \quad x - r - r \quad x + 1 \end{array}$$



$$\text{ب) } y = |x + r| + |x - r| \quad R = [r, +\infty)$$

$$|x - r| - |rx + r| \leq 1 \rightarrow \begin{array}{l} x < -1 \quad -1 \leq x \leq r \quad x > r \\ -x + r + rx + r \quad -x + r - rx - r \quad x - r - rx - r \end{array}$$

$$\Rightarrow (-\infty, r] \left[\frac{1}{r}, +\infty \right)$$

$$\text{جی) } y = [x] + [r - x] = r + [x] + [-x] \Rightarrow$$

$$\text{د) } y = rx - [x]$$

$$\begin{array}{ll} rx + r & -r \leq x < -1 \\ rx + 1 & -1 \leq x < 0 \\ rx & 0 \leq x < 1 \\ rx - 1 & 1 \leq x < r \\ r & x = r \end{array}$$

$$\sin x = 1 \rightarrow 1$$

$$\sin x = -1 \rightarrow r$$

$$\frac{-b}{ra} \rightarrow \frac{1}{r}$$

$$\sin x = 1 \rightarrow r$$

$$\sin x = 0 \rightarrow 1$$

$$\frac{-b}{ra} \rightarrow -\frac{1}{r}$$

$$\text{الف) } y = \sin^r x \sin x + 1 \quad R = \left[\frac{1}{r}, r \right]$$

$$\text{ب) } y = \sin^r x + \sin^r x + 1 \quad R = \left[-\frac{1}{r}, r \right]$$