۱ـ فیلوینی ۱۲ام سه شنبه

$yx - y = x^2 + x + 2 \to x^2 + x(1-y) + 2 + y = 0 \to$

$y \neq 1 \to x = \dfrac{-1+y \pm \sqrt{(1-y)^2 - 4(1)(2+y)}}{2} \xrightarrow{\Delta \geq 0} y^2 - 2y + 1 - 8 - 4y \geq 0$

$y^2 - 6y - 7 \geq 0 \to (y-7)(y+1) \geq 0$

$y = 1 \to x^2 = -3$ ت ن

$+ \quad -1 \quad - \quad 7 \quad +$ $\quad \boxed{y \geq 7 \text{ یا } y \leq -1}$

۲ـ

الف) $y = 2x^2 - 5x + 1 \to a > 0 \to x_{min} = -\dfrac{b}{2a} = \dfrac{5}{4} \to y_{min} = 2\left(\dfrac{25}{16}\right) - \dfrac{25}{4} + 1 = -\dfrac{17}{8}$

برد $\to \left[-\dfrac{17}{8}, +\infty\right)$

ب) $y = \sqrt{-x^2 + 6x - 5}$ $\quad x_{max} = -\dfrac{b}{2a} = -\dfrac{6}{-2} = 3 \to y_{max} = -9 + 18 - 5 = 4$

$a < 0$

برد $= \sqrt{[-\infty, 4]} = [0, 2]$

ج)

می دانیم برد عبارت چندجمله ای با بزرگترین توان فرد برابر R است

$\sqrt{R} = [0, +\infty)$

د) $\log(R)$ $\quad R_f = (-\infty, +\infty)$ " "

۳ـ

الف) $2yx - 6y = 3x + 1 \to 3x - 2yx = -6y - 1 \to x(3 - 2y) = -6y - 1$

$\to x = \dfrac{6y+1}{2y-3} \quad R_f = R - \left\{\dfrac{3}{2}\right\}$

ب) ① $y \geq 0$

② $y^2 = \dfrac{4x+1}{x-2} \to 4x + 1 = y^2 x - 2y^2 \to x(4 - y^2) = -2y^2 - 1$

$x = \dfrac{2y^2+1}{y^2-4} \quad y^2 - 4 \neq 0 \to y \neq \pm 2$

$\to [0, +\infty) - \{-2\}$

پ) $\dfrac{x^2+4}{\sqrt{x^2+3}} = \dfrac{x^2+3+1}{\sqrt{x^2+3}} = \dfrac{x^2+3}{\sqrt{x^2+3}} + \dfrac{1}{\sqrt{x^2+3}} = \sqrt{x^2+3} + \dfrac{1}{\sqrt{x^2+3}}$

می دانیم اگر $a \geq 0 \leftarrow a + \dfrac{1}{a} \geq 2$

$\sqrt{x^2+3} \xrightarrow[\text{به ازای } x=0]{min} \to \sqrt{0+3} = \sqrt{3} \to \sqrt{3} + \dfrac{1}{\sqrt{3}} = \dfrac{4\sqrt{3}}{3}$

$R_f = \left[\dfrac{4\sqrt{3}}{3}, +\infty\right)$

د) $x^2+4+\frac{1}{x^2+4}-4 \to \underbrace{x^2+4}_{a\geq 4}+\frac{1}{x^2+4} \to a+\frac{1}{a} \to min$ به ازای $x=0 \to 4+\frac{1}{4}=\frac{17}{4}$

$R_f=[\frac{1}{4},+\infty)$ $\qquad \to \frac{17}{4}-4=\frac{1}{4}$

۴-

$yx^2-y=3x^2+4 \to yx^2-3x^2=y+4 \to x^2(y-3)=y+4$

$\to x^2=\frac{y+4}{y-3}$

$\to \frac{y+4}{y-3}\geq 0$ $\qquad$ $+ \underset{-4}{0} - \underset{3}{\text{ن‌ت}} +$

$\boxed{y>3 \cup y\leq -4}$

$y=\frac{3\overset{\square}{x^2}+4}{x^2-1}$ $\qquad \square \to x^2 \to x^2\geq 0 \to x^2=0 \to y=\frac{4}{-1}=-4$

$\searrow x^2=\infty \to y=\frac{3\infty+4}{\infty-1}=\frac{3\infty}{\infty}=3$

$x^2-1=0 \to x^2=1 \to$ مخرج نمی‌تواند صفر شود $\to R_f=(-\infty,-4]\cup(3,+\infty)$

۵-

$y=\frac{2\sin x-1}{\sin x+3} \to y\sin x+3y=2\sin x-1 \to \sin x(2-y)=1+3y$

$\to \sin x=\frac{1+3y}{2-y} \to -1\leq \frac{1+3y}{2-y}\leq 1 \to \frac{1+3y}{2-y}\geq -1$

$\to \frac{1+3y}{2-y}+1\geq 0 \to \frac{1+3y+2-y}{2-y}=\frac{2y+3}{2-y}\geq 0 \to$ $\quad - \underset{-\frac{3}{2}}{|} + \underset{2}{|} -$

$\frac{1+3y}{2-y}-1\leq 0 \to \frac{1+3y-2+y}{2-y}=\frac{4y-1}{2-y}\leq 0$ $\qquad$ ① $\to \boxed{-\frac{3}{2}\leq y<2}$

$- \underset{\frac{1}{4}}{|} + \underset{2}{|} -$ $\qquad$ ② $\boxed{\begin{matrix} y>2 \\ y\leq \frac{1}{4}\end{matrix}}$

① ∩ ② $= -\frac{3}{2}\leq y\leq \frac{1}{4}$

راه۲ $\to \square=\sin x \quad -1\leq \sin x\leq 1 \to \sin x=1 \to y=\frac{2-1}{1+3}=\frac{1}{4}$

$\searrow \sin x=-1 \to y=\frac{-2-1}{-1+3}=-\frac{3}{2}$

$\sin x+3=0 \to \sin x=-3 \to$ مخرج صفر نمی‌شود $\to -\frac{3}{2}\leq y\leq \frac{1}{4}$

۶-

$f(x)=\frac{x^2-x+\frac{1}{4}}{x^2-x+1}$ $\qquad D_f=R-\{$ریشه مخرج$\}$

$x^2-x+1=0 \to \Delta<0 \to D_f=R$

$yx^2-yx+y=x^2-x+\frac{1}{4} \to x^2(y-1)+x(-y-1)+y-\frac{1}{4}=0$

$x=\frac{1+y\pm\sqrt{(1+y)^2-4(y-1)(y-\frac{1}{4})}}{2(y-1)} \to \Delta\geq 0 \to$ $\qquad$ ادامه صفحه بعد

$y^2+2y+1-4(y^2-\frac{5}{4}y+\frac{1}{4})>0 \to y^2+2y+1-4y^2+20y-1>0$

$-3y^2+22y>0 \to y(-3y+22)>0$

$- \quad 0 \quad + \quad \frac{22}{3} \quad - \quad \to 0<y<\frac{22}{3}$

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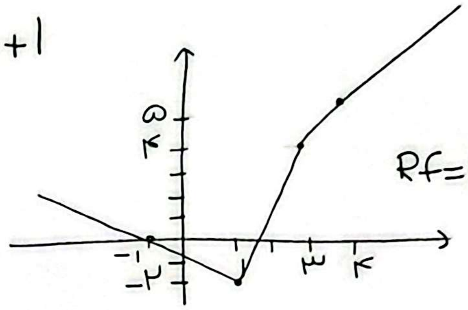
الف)

$1 \qquad 3$

$2x-2-x+3=x+1$

$2x-2+x-3=3x-5$

$-2x+2+x-3=-x-1$



$R_f=[-2,+\infty)$

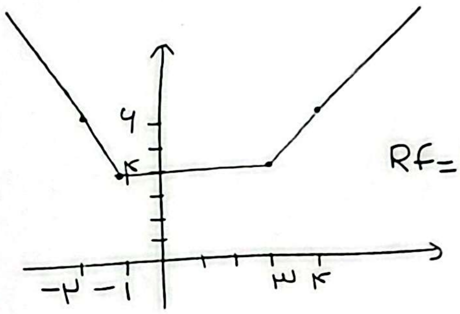
ب)

$-1 \qquad 3$

$2x-2$

$-x+3+x+1=4$

$-x+3-x-1=-2x+2$



$R_f=[4,+\infty)$

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$-1 \qquad 2$

$x-2-2x-2=-x-4\leq 1$

$-x\leq 5 \quad x\geq -5$ ③

$-x+2-2x-2=-3x\leq 1 \to x\geq -\frac{1}{3}$

$-x+2+2x+2=x+4\leq 1$

$\boxed{x\leq -3}$ ①

$\boxed{-\frac{1}{3}\leq x\leq 2}$ ②

$① \cup ② \cup ③ = [-\infty,-3]\cup[-\frac{1}{3},+\infty)$

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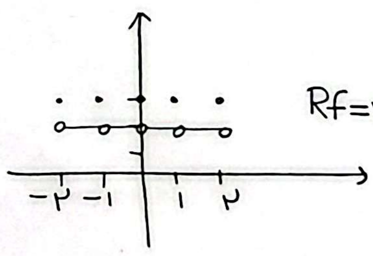
الف) $[x]+[3-x]=[x]+[-x]+3 \to$

$x\in Z \to [x]+[-x]=0 \to y=3$

$x\notin Z \to [x]+[-x]=-1 \to y=2$



$R_f=\{2,3\}$

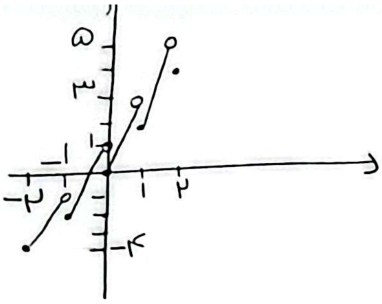
ب) $y=3x-[x] \to -2\leq x<-1 \to [x]=-2 \to y=3x+2$

$-1\leq x<0 \to [x]=-1 \to y=3x+1$

$0\leq x<1 \to [x]=0 \to y=3x$

$1\leq x<2 \to [x]=1 \to y=3x-1$

$x=2 \to [x]=2 \to y=3x-2$



$R_f=[-4,5)$

۱۰-

الف) $y=\sin^2 x-\sin x+1 \rightarrow \square=\sin x \rightarrow -1\leq \sin x\leq 1$

$\sin x=-1 \rightarrow y=1+1+1=3$

$\sin x=1 \rightarrow y=1-1+1=1$

$\sin x=-\frac{b}{2a}=\frac{1}{2} \rightarrow y=\frac{1}{4}-\frac{1}{2}+1=\frac{3}{4}$

$\rightarrow R_f=[\frac{3}{4},3]$

ب) $y=\sin^4 x+\sin^2 x+1 \quad \square=\sin^2 x \quad 0\leq \sin^2 x\leq 1$

$\sin^2 x=0 \rightarrow y=1$

$\sin^2 x=1 \rightarrow y=3$

$\sin^2 x=-\frac{b}{2a}=\frac{-1}{2}$ غ ق ق

$\rightarrow R_f=[1,3]$