

$$y = \frac{x^2 + x + 2}{x - 1}$$

$$x^2 + x + 2 = yx - y \quad x^2 + (1 - y)x + 2 + y = 0$$

$$x = \frac{y - 1 \pm \sqrt{(1 - y)^2 - 4y - 8}}{2}$$

$$(1 - y)^2 - 4y - 8 \geq 0 \quad (y - 5)(y + 1) \geq 0$$

$$R_f = R - (-1, 5) \checkmark$$

الف) $y = 2x^2 - 5x + 1$

$$\text{ext.} \left| \frac{-b}{2a} \right|$$

$$\text{ext.} \left| \frac{\Delta}{4a} \right|$$

$$y = \left[\frac{-15}{4}, +\infty \right)$$

ب) $y = \sqrt{-x^2 + 4x - 5}$

$$\text{ext.} \left| \frac{3}{4} \right|$$

$$y = \sqrt{(-\infty, 4)} = [0, 2]$$

ج) $y = \sqrt{x^4 - 4x^2 + 4x + 1}$

$$y = [0, +\infty) \checkmark$$

۱) $y = (x^3 - 4x^2 + 17x + 4)$

$$x^3 - 4x^2 + 17x + 4 > 0 \quad (x - 2)^3 > 5x - 12$$

$$y = R \checkmark$$

الف) $y = \frac{3x + 1}{2x - 4}$

$$R - \left\{ \frac{3}{2} \right\}$$

ب) $y = \sqrt{\frac{3x + 1}{2x - 4}}$

$$y = [0, +\infty) - \left\{ \frac{3}{2} \right\}$$

ج) $y = \frac{x^2 + 4}{\sqrt{x^2 + 3}}$

$$\frac{x^2 + 4}{\sqrt{x^2 + 3}} = \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}}$$

$$y = \left[\sqrt{3 + \frac{1}{\sqrt{3}}}, +\infty \right)$$

۲) $y = x^2 + \frac{1}{x^2 + 4} + 4 - 4$

$$x^2 + 4 + \frac{1}{x^2 + 4} - 4 \xrightarrow{x \rightarrow \infty} \frac{1}{4} - 4$$

$$y = \left[\frac{1}{4}, +\infty \right) \checkmark$$

$$y = \frac{3x^2 + 4}{x^2 - 1}$$

$$x = \frac{3y^2 + 4}{y^2 - 1}$$

$$y^2 = -\frac{-x - 4}{x - 3} = \frac{x + 4}{x - 3}$$

روند اول

$$y = \pm \sqrt{\frac{x + 4}{x - 3}}$$

$$\frac{-4}{+} \quad \frac{3}{-1+}$$

$$y = (-\infty, -4] \cup [3, +\infty)$$

روند دوم

$$0 < x^2 < +\infty$$

$$x^2 = 0 \rightarrow \frac{0 + 4}{0 - 1} = -4$$

$$x^2 = +\infty \rightarrow \frac{3}{1} = 3$$

$$(-\infty, -4] \cup (3, +\infty)$$

چون $+\infty$ باز است $\checkmark$

چون مربع منفی ندارد

سوی خارج رشته ها جواب است

$$y = \frac{r \sin \alpha - 1}{\sin \alpha + r}$$

$$\begin{aligned} \sin \alpha = -1 &\rightarrow \frac{-r-1}{-1+r} = \frac{r}{r} \\ \sin \alpha = 1 &\rightarrow \frac{r-1}{r} = \frac{1}{r} \end{aligned}$$

$$r = \left[\frac{-r}{r} \div \frac{1}{r} \right]$$

$$y = \frac{r \sin \alpha - 1}{\sin \alpha + r}$$

$$r \sin \alpha - 1 = y \sin \alpha + ry$$

$$\sin \alpha = \frac{ry+1}{r-y}$$

$$-1 \leq \frac{ry+1}{r-y} \leq 1$$

$$\frac{ry+r}{r-y} \geq 0 \quad \frac{-r}{r-y} \leq 0 \quad \frac{ry-1}{r-y} \leq 0 \quad \frac{1}{r-y} \geq 0$$

$$f(x) = \frac{x^2 - x + \frac{1}{r}}{x^2 - x + 1}$$

$$x^2 - x + 1 \neq 0 \quad D_f = \mathbb{R}$$

$$y = \frac{x^2 - x + \frac{1}{r}}{x^2 - x + 1}$$

$$x^2 - x + \frac{1}{r} = yx^2 - yx + y$$

$$yx^2 - x^2 - yx + x + y - \frac{1}{r} = 0$$

$$(y-1)x^2 - (y-1)x + y - \frac{1}{r} = 0$$

$$x = \frac{y-1 \pm \sqrt{(y-1)^2 - 4(y-\frac{1}{r})(y-1)}}{2(y-1)}$$

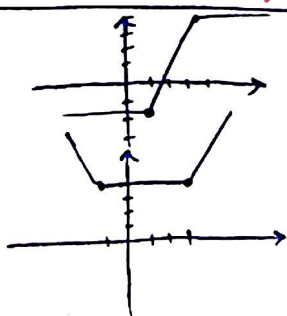
$$(y-1)^2 - 4(y-\frac{1}{r})(y-1) \geq 0 \quad (y-1)(y-1-4y+4) \geq 0 \quad (y-1)(-3y+3) \geq 0$$

$$(y-1)(3y) \leq 0 \quad \frac{0}{+} \quad \frac{1}{-} \quad \frac{1}{+}$$

شرط استعداده داشتن، نخواهد بود ضریب x^2 با عدد یک است
 $y \neq 1$ بودن و برابر یک =
 $0 < y < 1$
 $y \in [0, 1)$

$$y = |x-2| - |x-3|$$

$$x=1 \quad x=3$$



$$y = [-2, 4]$$

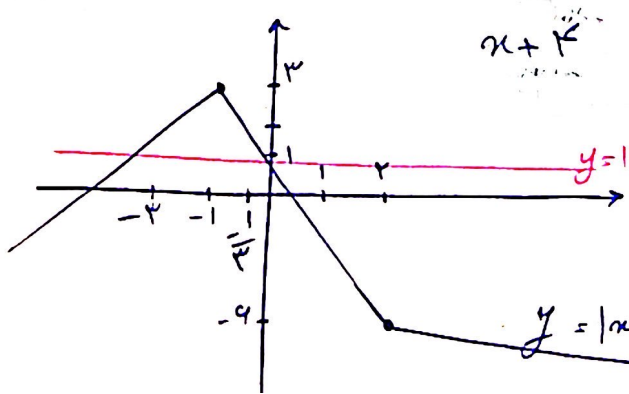
$$[-2, +\infty)$$

$$y = [3, +\infty)$$

$$y = |x-3| + |x+1|$$

$$|x-2| - |x+2| \leq 1$$

$$\begin{aligned} -x+2+x+2 &= 4 \\ x-2-x-2 &= -4 \\ x-2-x+2 &= 0 \end{aligned}$$



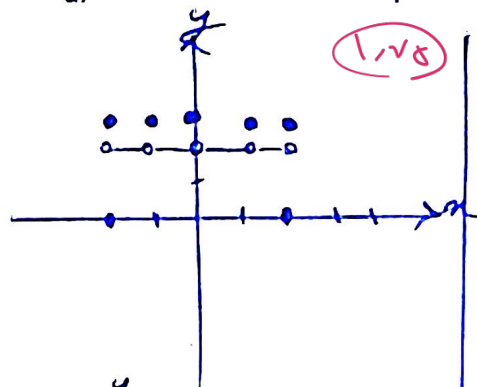
$$(-\infty, -2] \cup [\frac{1}{r}, +\infty)$$

$$y = |x-2| - |x+2|$$

$$y = [x] + [x - \pi] = [x] + [-x] + \pi$$

$$[x] + [-x] \begin{cases} 0 & x \in \mathbb{Z} \\ -1 & x \notin \mathbb{Z} \end{cases} \xrightarrow{+\pi} \begin{cases} \pi & x \in \mathbb{Z} \\ \pi - 1 & x \notin \mathbb{Z} \end{cases}$$

$$Ry = [\pi, \pi]$$



$$y = \pi x - [x] \begin{cases} \pi x + \pi - \pi x < -1 \rightarrow [x] = -\pi \\ \pi x + 1 - \pi x < 0 \rightarrow [x] = -1 \\ \pi x < 0 < \pi x + 1 \rightarrow [x] = 0 \\ \pi x - 1 < \pi x < \pi x + 1 \rightarrow [x] = 1 \\ \pi x - \pi < \pi x - \pi + 1 \rightarrow [x] = \pi \end{cases}$$

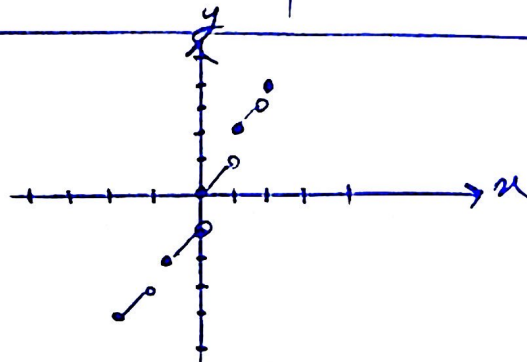
$$\pi x + 1 - \pi x < 0 \rightarrow [x] = -1$$

$$\pi x < 0 < \pi x + 1 \rightarrow [x] = 0$$

$$\pi x - 1 < \pi x < \pi x + 1 \rightarrow [x] = 1$$

$$\pi x - \pi < \pi x - \pi + 1 \rightarrow [x] = \pi$$

$$Ry = [-2, 8)$$



$$y = \sin^2 x - \sin x + 1 \begin{cases} \sin x = 1 \rightarrow 1 - 1 + 1 = 1 \\ \sin x = -1 \rightarrow 1 + 1 + 1 = 3 \\ \frac{-b}{2a} = \frac{1}{2} \rightarrow \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4} \end{cases}$$

$$\sin x = -1 \rightarrow 1 + 1 + 1 = 3$$

$$\frac{-b}{2a} = \frac{1}{2} \rightarrow \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}$$

$$Ry = [\frac{3}{4}, 3]$$

$$y = \sin^2 x + \sin^2 x + 1 \begin{cases} \sin x = 0 \rightarrow 0 + 0 + 1 = 1 \\ \sin x = 1 \rightarrow 1 + 1 + 1 = 3 \\ \frac{-b}{2a} = \frac{-1}{2} \rightarrow \frac{1}{4} + \frac{1}{4} + 1 = \frac{5}{4} \end{cases}$$

$$\sin x = 1 \rightarrow 1 + 1 + 1 = 3$$

$$\frac{-b}{2a} = \frac{-1}{2} \rightarrow \frac{1}{4} + \frac{1}{4} + 1 = \frac{5}{4}$$

$$Ry = [1, 3]$$