

۱۹، ۵ آفرین!

آیة الله العظمى
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تألیف

$$y = \frac{x^2 + x + 1}{x - 1}$$

$$yx - y = x^2 + x + 1$$

$$x^2 + (1 - y)x + 1 + y = 0$$

$$\Delta \geq 0 \quad (1 - y)^2 - 4(1 + y) \geq 0$$

$$1 + y^2 - 2y - 4 - 4y = 0$$

$$y^2 - 4y - 3 \geq 0$$

$$y^2 - 4y - 3 = 0$$

$$\frac{4 + \sqrt{40}}{2} = y$$

$$\frac{4 - \sqrt{40}}{2} = -1$$

$$\frac{-1 \pm \sqrt{16}}{2}$$

$$(-\infty, -1] \cup [7, +\infty)$$

① ②

الف) $y = 2x^2 - 5x + 1$ $x_{\min} = \frac{5}{4}$ $y = \left[\frac{17}{8}, +\infty\right)$

$$y = -\frac{17}{8}$$

ب) $y = \sqrt{-x^2 + 4x - 5}$

$$\frac{-4}{-2} = 2 \quad y = 4, \quad (-\infty, 4] \rightarrow [0, 2]$$

ج) $y = \sqrt{x^2 - 3x^2 + 4x + 1}$ $\rightarrow [0, +\infty)$

د) $\log(x^2 - 4x^2 + 11x + 4) \rightarrow (-\infty, +\infty)$

الف) $y = \frac{3x + 1}{2x - 4}$ $\mathbb{R} - \left\{\frac{3}{2}\right\}$

ب) $y = \sqrt{\frac{4x + 1}{x - 2}}$ $\mathbb{R} - \{2\} \rightarrow [0, +\infty) - \{2\}$

ج) $y = \frac{x^2 + 1}{\sqrt{x^2 + 3}}$ $\frac{x^2 + 3 + 1}{\sqrt{x^2 + 3}} = \frac{x^2 + 3}{\sqrt{x^2 + 3}} + \frac{1}{\sqrt{x^2 + 3}} = \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}}$

د) $x^2 + \frac{1}{x^2 + 3} + 4 - 4$

الف) $x^2 + 3 + \frac{1}{x^2 + 3} - 4$

ب) $\left[\frac{17}{8}, +\infty\right) - 4 \rightarrow \left[\frac{1}{8}, +\infty\right)$

$$y = \frac{3x^2 + 4}{x^2 - 1}$$

$$yx^2 - y = 3x^2 + 4$$

$$yx^2 - 3x^2 = y + 4$$

$$x^2(y - 3) = y + 4$$

$$x^2 = \frac{y+4}{y-3} \quad x = \pm \sqrt{\frac{y+4}{y-3}}$$

$$\frac{-4}{+} \cdot \frac{3}{-} = +$$

$$(-\infty, -4] \cup (3, +\infty)$$

روش اول: $(-\infty, -4] \cup (3, +\infty)$
 خارج صفر شود
 مخرج = $\frac{4}{-1} = -4$
 مخرج صفر شود $x^2 = +\infty = 3$

$$y = \frac{2\sin x - 1}{\sin x + 3}$$

$$2\sin x - 1 = y\sin x + 3y$$

$$(2-y)\sin x = 3y+1$$

$$\sin x = \frac{3y+1}{2-y}$$

$$-1 \leq \frac{3y+1}{2-y} \leq 1$$

روش دوم: $\sin x = 1$ $\sin x = -1$
 $\frac{3y+1}{2-y} = 1 \rightarrow [-\frac{3}{2}, \frac{1}{2}]$
 $\frac{3y+1}{2-y} = -1 \rightarrow [-\frac{3}{2}, \frac{1}{2}]$

$$\frac{3y+1}{2-y} \geq -1$$

$$\frac{3y+1+2-y}{2-y} \geq 0$$

$$\frac{2y+3}{2-y} \geq 0$$

$$\frac{2y+3}{2-y} \leq 1$$

$$\frac{2y+3-2+y}{2-y} \leq 0$$

$$\frac{y+1}{2-y} \leq 0$$

$$y = \frac{x^2 - x + \frac{1}{4}}{x^2 - x + 1}$$

$$yx^2 - xy + y = x^2 - x + \frac{1}{4}$$

$$(y-1)x^2 - (y-1)x + y - \frac{1}{4} = 0$$

دامنه: $x^2 - x + 1 \neq 0$ همیشه دارد $Df = R$
 $1 - f(1) = \Delta < 0$ است

$$\Delta \geq 0$$

$$(y-1)^2 - 4(y-1)(y-\frac{1}{4}) \geq 0$$

$$(y-1)^2 - 4y^2 + y + 1 \geq 0$$

$$(y-1)(y-1-4y+1) \geq 0$$

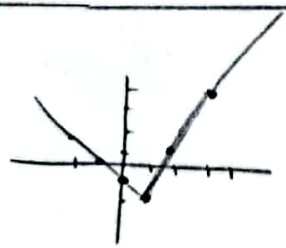
$$(y-1)(-3y) \geq 0$$

$$0 \leq y \leq 1$$

$$R = [0, 1]$$

الف) $y = |2x-2| - |x-3|$

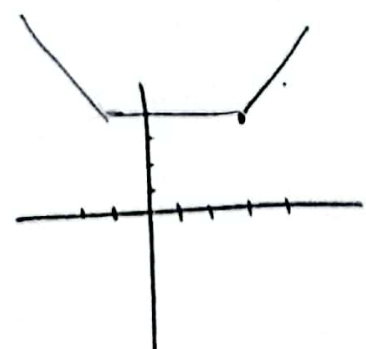
$2x+2-x-2$	$x-2+x-3$	$2x-2-x+3$
x	$x-5$	$x+1$



بردار: $[-2, +\infty)$

ب) $y = |x-3| + |x+1|$

$-x+3-x-1$	$-x+3+x+1$	$x-3+x+1$
$-2x+2$	4	$2x-2$



بردار: $[4, +\infty)$

$$|x-2| - |2x+2| \leq 1$$

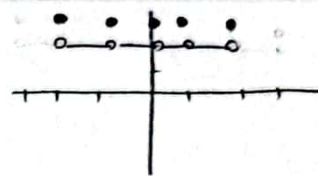
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2

$x < -1$	$-1 \leq x < 2$	$x \geq 2$
$-x+2+2x+2$	$-x+2-2x-2$	$x-2-2x-2$
$x+4 \leq 1$	$-3x \leq 1$	$-x-4 \leq 1$
$x \leq -3$	$-x \leq \frac{1}{3}$	$-x \leq 5 \Rightarrow x \geq -5$
$x \leq -3$	$x \geq -\frac{1}{3}$	$x \geq -5$
$x \leq -3$	$-\frac{1}{3} \leq x < 2$	$x \geq 2$

$$(-\infty, -3] \cup [-\frac{1}{3}, +\infty)$$

$\text{حيث } [x] + [1-x] \quad [x] + [-x] + 1$
 $x \in \mathbb{Z} \quad 0+1=1$
 $x \notin \mathbb{Z} \quad -1+1=0$

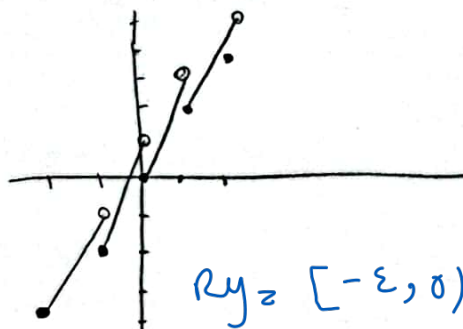


9

1, 0

ب) $y = 1 - [x]$

$[0, 1)$	$1 - x$
$[1, 2)$	$1 - x - 1$
2	$1 - x - 2$
$[-1, 0)$	$1 - x + 1$
$[-2, -1)$	$1 - x + 2$



$$Ry = [-1, 0)$$

$y = \sin^2 x - \sin x + 1$
 $1+1+1$
 $\text{max} = 1$
 $\text{min} = 1$

$$[\frac{1}{4}, 1]$$

$y = \sin^2 x + \sin^2 x + 1$
 $\text{max} = 2$
 $\text{min} = 0$

$$[0, 2]$$

10

11