

$$y = \frac{x^2 + x + 2}{x - 1} \Rightarrow yx - y = x^2 + x + 2 \Rightarrow x^2 + x(1 - y) + (y + 2) = 0 \Rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\Delta \geq 0 \Rightarrow (1 - y)^2 - 4(y + 2) \geq 0 \Rightarrow y^2 - 4y - 7 \geq 0 \Rightarrow R_f = (-\infty, -1] \cup [7, +\infty)$$

$$y = 2\left(\frac{25}{14}\right) - \frac{25}{14} + 1 = -\frac{25}{14} + 1 = -\frac{11}{14} \quad R_f = \left[-\frac{11}{14}, +\infty\right)$$

$$الف) y = 4x^2 - 5x + 1 \xrightarrow{a \geq 0} \begin{cases} x_{min} = \frac{-b}{2a} = \frac{5}{8} \\ y_{min} = \frac{341}{14} \end{cases} \Rightarrow R_f = \left[\frac{341}{14}, +\infty\right)$$

$$ب) y = \sqrt{-x^2 + 4x - 5} \xrightarrow{a < 0} \begin{cases} x_{max} = \frac{-b}{2a} = 2 \\ y_{max} = 4 \end{cases} \Rightarrow R_f = \sqrt{(-\infty, 4]} = [0, 2] \checkmark$$

ج) $y = \sqrt{x^2 - 3x^2 + 4x + 1} \rightarrow R_f = [0, +\infty)$ اگر این عبارت زیر رادیکال نباشد بردش R بود اما که زیر رادیکال است

د) $\log(x^3 - 4x^2 + 7x + 4)$ چون بزرگترین فرد است $R_f = (-\infty, +\infty)$

ه) $y = \frac{3x + 1}{2x - 4} \xrightarrow{\text{همگامی}} R_f = \mathbb{R} - \left\{\frac{3}{2}\right\}$ دیت کند! معانب قائم

و) $y = \sqrt{\frac{4x + 1}{x - 2}} \xrightarrow{\text{همگامی}} R_f = \sqrt{\mathbb{R} - \{2\}} \Rightarrow R_f = [0, +\infty) - \{2\} \checkmark$

ز) $y = \frac{x^2 + 4}{\sqrt{x^2 + 3}} \Rightarrow y = \frac{x^2 + 3 + 1}{\sqrt{x^2 + 3}} = \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}} \xrightarrow{\text{کثرین مقدار که می تواند داشته باشد}} \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}} \Rightarrow R_f = \left[\frac{4}{\sqrt{3}}, +\infty\right)$

ح) $y = x^2 + \frac{1}{x^2 + 4} \Rightarrow y = x^2 + 4 + \frac{1}{x^2 + 4} - 4 \xrightarrow{\text{کثرین مقدار که می تواند داشته باشد}} 4 + \frac{1}{4} - 4 = \frac{1}{4} \Rightarrow R_f = \left[\frac{1}{4}, +\infty\right)$

ادش لگی:

$$y = \frac{3x^2 + 4}{x^2 - 1} \xrightarrow{\text{مخرج صفر شود}} \begin{cases} y = -4 \\ y = 3 \end{cases} \Rightarrow R_f = (-\infty, -4] \cup [3, +\infty) \checkmark$$

روش اصلی:

$$y = \frac{3x^2 + 4}{x^2 - 1} \Rightarrow yx^2 - y = 3x^2 + 4 \Rightarrow yx^2 - 3x^2 = y + 4 \Rightarrow x^2(y - 3) = y + 4 \Rightarrow x = \sqrt{\frac{y + 4}{y - 3}} \Rightarrow \frac{y + 4}{y - 3} \geq 0 \Rightarrow \frac{-4}{y - 3} \geq 0 \Rightarrow R_f = (-\infty, -4] \cup [3, +\infty) \checkmark$$

ادش لگی:

$$y = \frac{2 \sin x - 1}{\sin x + 3} \xrightarrow{\text{مخرج صفر شود}} \begin{cases} y = \frac{3}{4} \\ y = \frac{1}{4} \end{cases} \Rightarrow R_f = \left[\frac{1}{4}, \frac{3}{4}\right] \checkmark$$

روش اصلی:

$$y = \frac{2 \sin x - 1}{\sin x + 3} \Rightarrow y \sin x + y = 2 \sin x - 1 \Rightarrow \sin x(2 - y) = y + 1 \Rightarrow \sin x = \frac{y + 1}{2 - y} \Rightarrow -1 \leq \frac{y + 1}{2 - y} \leq 1 \Rightarrow \begin{cases} y + 1 \geq y - 2 \Rightarrow y \geq \frac{3}{4} \\ y + 1 \leq y - 2 \Rightarrow y \leq \frac{1}{4} \end{cases} \Rightarrow R_f = \left[\frac{1}{4}, \frac{3}{4}\right] \checkmark$$

$$f(x) = \frac{x^2 - x + \frac{1}{4}}{x^2 - x + 1}$$

$$D_f \rightarrow x^2 - x + 1 \neq 0 \Rightarrow \Delta < 0 \Rightarrow \min \Rightarrow D_f = \mathbb{R}$$

$$x^2 y - x y + y = x^2 - x + \frac{1}{4} \Rightarrow (y-1)x^2 - (y-1)x + y - \frac{1}{4} = 0 \Rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} \Rightarrow \Delta \geq 0$$

$$(y-1)^2 - 4(y-\frac{1}{4})(y-1) \geq 0 \Rightarrow (y-1)(y-1-(y+1)) \geq 0 \Rightarrow (y-1)(-2y) \geq 0 \Rightarrow y \leq 1$$

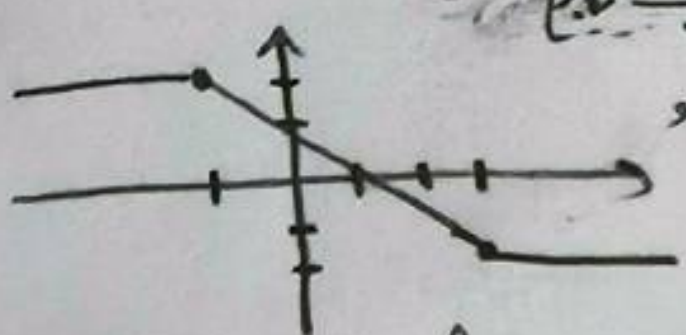
$$R_f = [0, 1) \quad \checkmark$$

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$$y = |x-3| + |x+1|$$

ب. تیر گذری است!



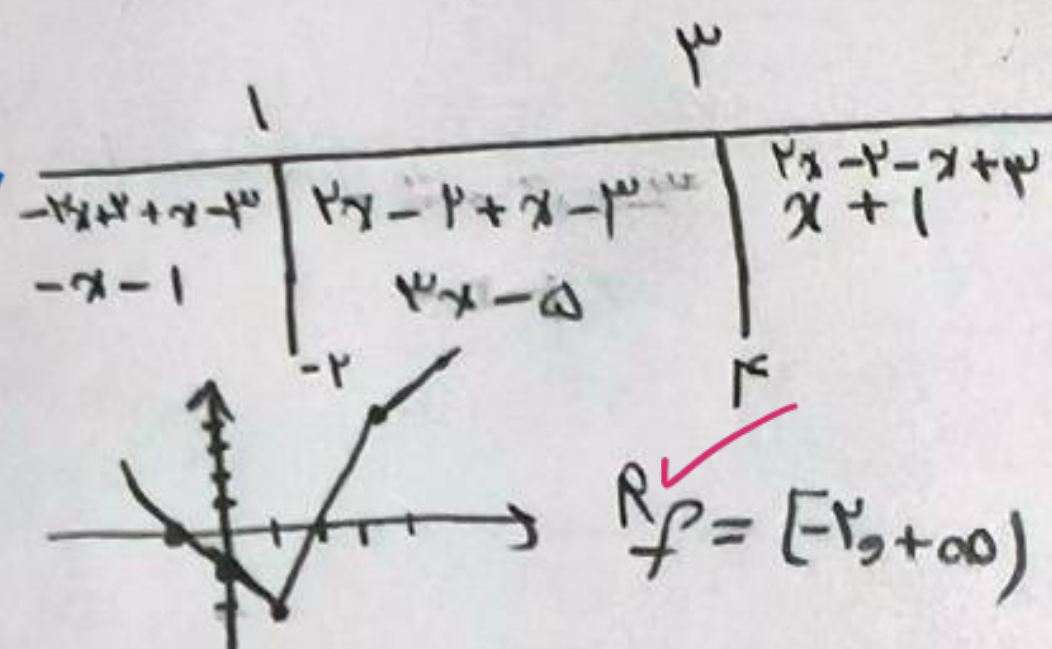
مقادیر ریشه ها = ۲ = ثابت = ۲
ریشه ها = ۱ و ۳ = پایین و بالایی تیر

$$R_f = [-2, 2]$$

$$R_f = [-2, +\infty)$$

$$y = |2x-1| - |x-3|$$

الف

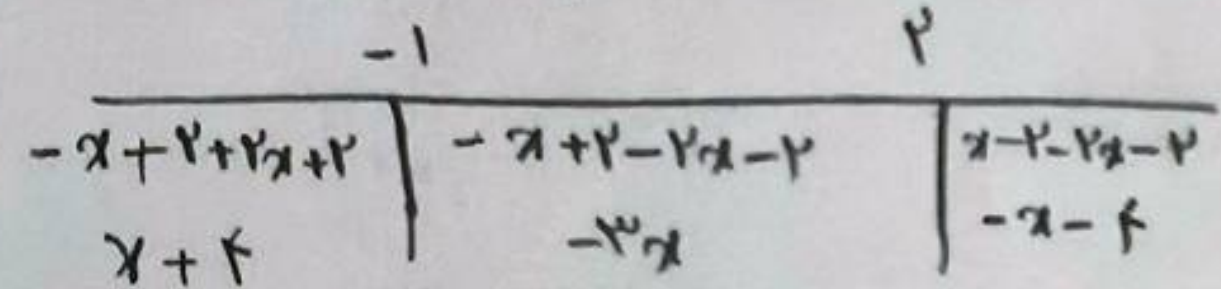


$$R_f = [-2, +\infty)$$

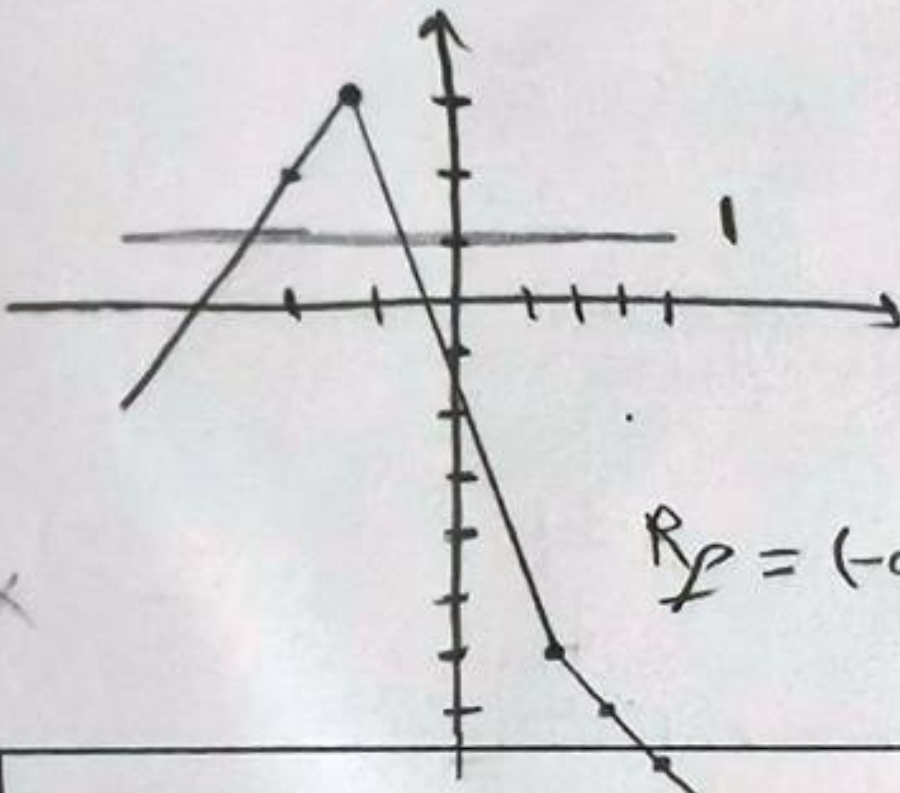
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$$y = |x-2| - |2x+2|$$



$x+2 \leq 1 \Rightarrow x \leq -1$
 $x+2 > 1 \Rightarrow x > -1$
 $-x+2 \leq -2 \Rightarrow x \geq 4$
 $-x+2 > -2 \Rightarrow x < 4$



$$R_f = (-\infty, -3] \cup [\frac{1}{2}, +\infty)$$

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$$y = 3x - [x]$$

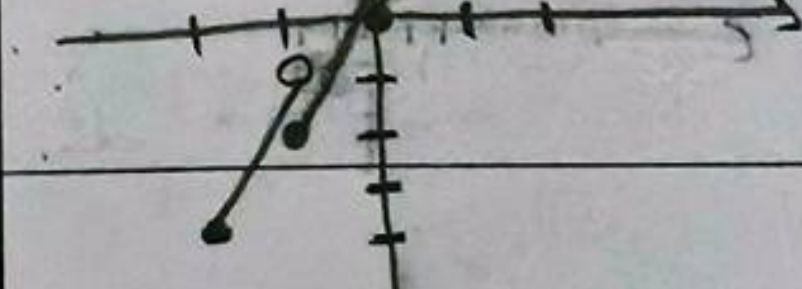
$$-2 \leq x < -1 \Rightarrow [x] = -2 \Rightarrow y = 3x + 2$$

$$-1 \leq x < 0 \Rightarrow [x] = -1 \Rightarrow y = 3x + 1$$

$$0 \leq x < 1 \Rightarrow [x] = 0 \Rightarrow y = 3x$$

$$1 \leq x < 2 \Rightarrow [x] = 1 \Rightarrow y = 3x - 1$$

$$x = 2 \Rightarrow y = 3x - 2$$



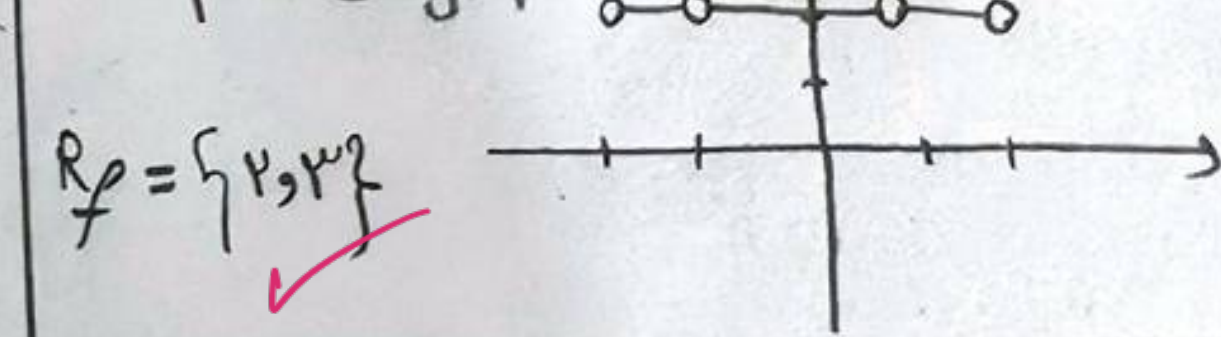
$$R_y = [-2, 0)$$

ب.

$$y = [x] + [3-x] \Rightarrow y = [x] + [-x] + 3$$

الف

$$y' = [x] + [-x] \begin{cases} x \in \mathbb{Z} \Rightarrow y' = 0 \\ x \notin \mathbb{Z} \Rightarrow y' = -1 \end{cases}$$



$$R_f = [2, 3]$$

۱, ۲

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$$y = \sin^2 x + \sin^2 x + 1$$

ب.

$$y = \sin^2 x - \sin x + 1$$

الف

$$R_f = [\frac{11}{4}, 3] \quad \checkmark$$

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