

۱۹، ۷۵ آزمون ضمیمه نوشتی!

کلیف شما ۳

نکته ۱

$$y = \frac{x^2 + x + 2}{x - 1} \rightarrow yx - y = x^2 + x + 2 \rightarrow x^2 + x(1 - y) + 2 + y = 0$$

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$$\Delta = b^2 - 4ac = (1 - y)^2 - 4(2 + y) = 1 + y^2 - 2y - 8 - 4y \rightarrow y^2 - 6y - 7$$

$$x = \frac{y - 1 \pm \sqrt{y^2 - 6y - 7}}{2} \rightarrow \Delta \geq 0 \rightarrow y^2 - 6y - 7 \geq 0$$

$$(y - 7)(y + 1) \geq 0$$

$$\begin{array}{c|c|c} -1 & & 7 \\ \hline + & - & + \end{array}$$

$$R_f \rightarrow (-\infty, -1] \cup [7, +\infty) \checkmark$$

الف) $y = 2x^2 - 8x + 1 \rightarrow \begin{cases} \frac{b}{2a} = \frac{4}{2} \\ \frac{-17}{4} \end{cases} \rightarrow R_f = \left[\frac{-17}{4}, +\infty \right)$

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$$\rightarrow 2\left(\frac{4}{2}\right)^2 - 8\left(\frac{4}{2}\right) + 1 = \frac{-17}{4}$$

دارد $\min$

ب) $y = \sqrt{-x^2 + 6x - 8} \rightarrow \begin{cases} \frac{b}{2a} = \frac{3}{-1} = -3 \\ 4 \end{cases} \rightarrow R_f = \sqrt{(-\infty, 4]} = [0, 2]$

$$-(3)^2 + 6(3) - 8 = 4$$

دارد $\max$

ج) $y = \sqrt{x^2 - 3x^2 + 4x + 1} \rightarrow R_f = [0, +\infty)$

به برد عبارت زیر را نگاه
و نتایج زیر را بدست آورده
منتهی به ۰

د) $y = \log(x^2 - 2x^2 + 7x + 4) \rightarrow R_f = (-\infty, +\infty) = \mathbb{R}$

برد عبارت $\mathbb{R}$ است.

الف) $y = \frac{3x+1}{2x-2} \rightarrow 2xy - 2y = 3x+1 \rightarrow 3x - 2xy + 2y + 1 = 0$

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$$\rightarrow x(3 - 2y) + 2y + 1 = 0 \rightarrow x = \frac{2y+1}{2y-3} \rightarrow 2y-3 \neq 0 \rightarrow y \neq \frac{3}{2}$$

$$R_f \rightarrow \mathbb{R} - \left\{ \frac{3}{2} \right\} \checkmark$$

ب) $y = \sqrt{\frac{4x+1}{x-2}} \rightarrow R_f = [0, +\infty) - \{2\} \checkmark$

$$y^2 = \frac{4x+1}{x-2} \rightarrow y^2x - 2y^2 = 4x+1 \rightarrow 4x - y^2x = -2y^2 - 1 \rightarrow$$

$$x(4 - y^2) = -2y^2 - 1 \rightarrow x = \frac{2y^2+1}{y^2-4} \rightarrow \boxed{y > 0} \quad \boxed{y \neq 2}$$

$$c) y = \frac{x^2 + 2}{\sqrt{x^2 + 3}} = \frac{x^2 + 3 + 1}{\sqrt{x^2 + 3}} = \frac{x^2 + 3}{\sqrt{x^2 + 3}} + \frac{1}{\sqrt{x^2 + 3}}$$

$$= \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}} \xrightarrow[n^2 = 0]{\text{کمترین مقدار در } x=0} \sqrt{3} + \frac{1}{\sqrt{3}} = \sqrt{3} + \frac{\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}$$

$$R_f = \left[\frac{4\sqrt{3}}{3}, +\infty \right)$$

$$d) y = x^2 + \frac{1}{x^2 + 4} \xrightarrow[n^2 = 0]{\text{کمترین مقدار در } x=0} 0 + \frac{1}{0 + 4} = \frac{1}{4} \rightarrow R_f = \left[\frac{1}{4}, +\infty \right)$$

$$y = \frac{3x^2 + 4}{x^2 - 1}$$

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$$\text{روش اول} \rightarrow yx^2 - y = 3x^2 + 4 \rightarrow 3x^2 - yx^2 = -y - 4 \rightarrow x^2(3 - y) = -y - 4$$

$$\rightarrow x^2 = \frac{y + 4}{y - 3} \rightarrow x = \pm \sqrt{\frac{y + 4}{y - 3}} \rightarrow \frac{y + 4}{y - 3} \geq 0 \quad \begin{array}{c} -4 \quad 3 \\ + \quad | \quad - \quad | \quad + \end{array}$$

$$\rightarrow R_f = (-\infty, -4] \cup (3, +\infty)$$

$$\text{روش دوم} \rightarrow y = \frac{3x^2 + 4}{x^2 - 1} \xrightarrow[n^2 = 0]{\frac{4}{-1} = -4} \quad \xrightarrow[n^2 = +\infty]{3} \quad R_f = (-\infty, -4] \cup (3, +\infty)$$

چون این تابع هیچ دارد پس هیچ دو عدد بازه را نمی‌بوسیم.

$$y = \frac{2 \sin x - 1}{\sin x + 3}$$

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$$\text{روش اول} \rightarrow y \sin x + 3y = 2 \sin x - 1 \rightarrow 2 \sin x - y \sin x = 3y + 1$$

$$\sin x(2 - y) = 3y + 1 \rightarrow \sin x = \frac{3y + 1}{2 - y} \rightarrow -1 \leq \frac{3y + 1}{2 - y} \leq 1$$

$$\rightarrow \frac{3y + 1}{2 - y} \geq -1 \rightarrow \frac{3y + 1}{2 - y} + 1 \geq 0 \rightarrow \frac{3y + 3}{2 - y} \geq 0 \quad \begin{array}{c} -\frac{3}{2} \quad 2 \\ - \quad | \quad + \quad | \quad - \end{array}$$

$I \rightarrow \left[-\frac{3}{2}, 2 \right)$

$$\rightarrow \frac{3y + 1}{2 - y} \leq 1 \rightarrow \frac{3y + 1}{2 - y} - 1 \leq 0 \rightarrow \frac{y - 1}{2 - y} \leq 0 \quad \begin{array}{c} 1 \quad 2 \\ - \quad | \quad + \quad | \quad - \end{array}$$

$II \rightarrow \left(-\infty, \frac{1}{2} \right] \cup (2, +\infty)$

$$I \cap II = \left[-\frac{3}{2}, \frac{1}{2} \right] \quad \text{جواب}$$

$\sin x \rightarrow y = \frac{r \sin x - 1}{\sin x + 3}$ $\xrightarrow{\sin x = 1} \frac{r-1}{1+3} = \frac{1}{4}$ نکته: $\sin x \in [-1, 1]$
 $\xrightarrow{\sin x = -1} \frac{-r-1}{-1+3} = \frac{-r}{2} \rightarrow R_f \rightarrow \left[\frac{-r}{2}, \frac{1}{4} \right]$ ✓

$f(x) = \frac{x^2 - x + \frac{1}{4}}{x^2 - x + 1} \rightarrow yx^2 - yx + y = x^2 - x + \frac{1}{4}$ (۵) - ۶

$\rightarrow x^2 - yx^2 - x + xy + \frac{1}{4} - y = 0 \rightarrow x^2(1-y) + x(y-1) + y + \frac{1}{4} = 0$

$x = \frac{1-y \pm \sqrt{(y-1)^2 - 4(1-y)(\frac{1}{4}-y)}}{2(1-y)} \rightarrow (y-1)^2 - 4(1-y)(\frac{1}{4}-y) \geq 0$

$\Delta \geq 0 \rightarrow y^2 + 1 - 2y - 1 + 4y - 1 \geq 0 \rightarrow 3y - 1 \geq 0 \rightarrow y \geq \frac{1}{3}$

$\rightarrow -3y^2 + 3y \geq 0 \rightarrow 3y^2 - 3y \leq 0 \rightarrow 3y(y-1) \leq 0$ + | - | +

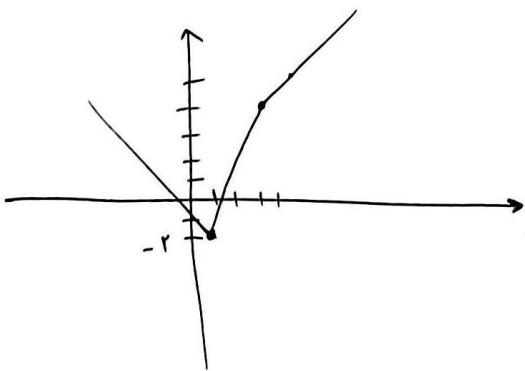
$R_f = \{0\} \rightarrow$ نکته: $[0, 1)$ نادرست است ✓ $[0, 1)$ ✓

$D_f \rightarrow x^2 - x + 1 \neq 0 \rightarrow \Delta < 0 \rightarrow 1 - 4(1)(1) = -3 \checkmark$

$\Delta < 0$ در $\mathbb{R}$ ندارد
 پس هیچگاه صفر نمی شود

$D_f = \mathbb{R}$

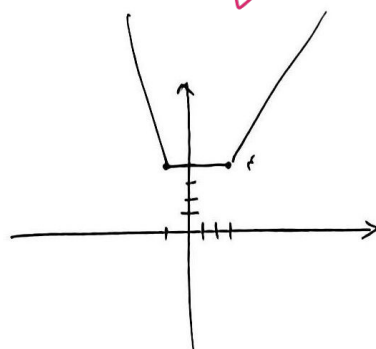
الف) $y = |2x-2| - |x-3|$



$2x-2-x+3 = x+1$	$2x-2-x+3 = x+1$	$2x-2-x+3 = x+1$
$x < 1$	$1 < x < 3$	$x > 3$
$-x-1$	$x-5$	$x+1$

(۲) - ۷

$R_f = [-2, +\infty)$ ✓



ب) $y = |x-3| + |x+1|$

$\begin{matrix} 3 & -1 \\ | & | \\ 4 & 4 \end{matrix}$

$R_f = [4, +\infty)$ ✓

$$|n-r| - |r_{n+r}| \leq 1$$

$$\begin{aligned} r-n - (-r_{n-r}) &\leq 1 & r-n - r_{n-r} &\leq 1 & n-r - r_{n-r} &\leq 1 \\ r-n + r_{n+r} &\leq 1 & -r_n &\leq 1 & -n-r &\leq 1 \rightarrow n \geq -r \\ n+r &\leq 1 & & & & \end{aligned}$$

III $n \leq -r$ II $n \geq \frac{-1}{r}$ I $n \geq -r$

$$R_f \rightarrow \left[\frac{-1}{r}, +\infty \right) \cup \left[-\infty, -r \right]$$

الف) $y = [n] + [r-n] = [n] + [-n] + r$

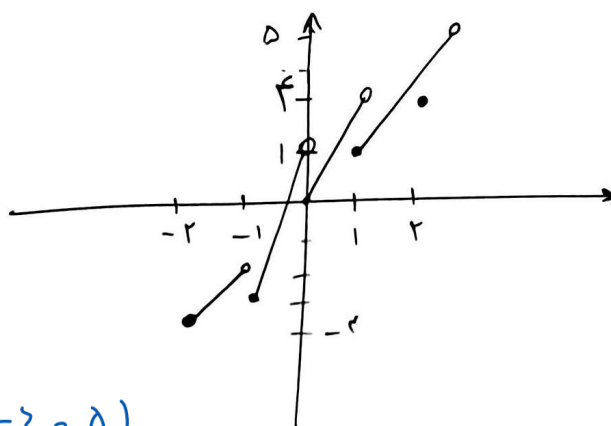
$[n] + [-n] \xrightarrow{n \in \mathbb{Z}} 0$
 $\xrightarrow{n \notin \mathbb{Z}} -1$

$Ry \subset \{r, r\}$

$[-r, r] \xrightarrow{r} \rightarrow$

ب) $y = r_n - [n]$

$[-r, -1) \rightarrow r_{n+r}$
 $[-1, 0) \rightarrow r_{n+1}$
 $[0, 1) \rightarrow r_{n+0} = r_n$
 $[1, r) \rightarrow r_{n-1}$
 $n = r \rightarrow r$



$$Ry \subset [-r, 0)$$

الف) $y = \sin^r n - \sin n + 1 \xrightarrow{\sin n = 1} 1 - 1 + 1 = 1$

$$\textcircled{2} - 1$$

$$R_f \rightarrow \left[\frac{r}{r}, r \right]$$

$$\begin{aligned} \xrightarrow{\sin n = -1} 1 + 1 + 1 &= 3 \\ \xrightarrow{\sin n = \frac{b}{ra} = \frac{1}{r}} \frac{1}{r} - \frac{1}{r} + 1 &= \left(\frac{r}{r} \right) \end{aligned}$$

ب) $y = \sin^r n + \sin^r n + 1 \xrightarrow{\sin^r n = 0} 0 + 0 + 1 = 1$

$$\xrightarrow{\sin^r n = 1} 1 + 1 + 1 = 3$$

$$\xrightarrow{\sin^r n = \frac{b}{ra} = \frac{1}{r}} \frac{1}{r} + \frac{1}{r} + 1 = \left(\frac{r}{r} \right)$$

$$R_f \rightarrow [1, 3]$$