

$$y = \frac{x^2 + x + 2}{x - 1} \Rightarrow yx - y = x^2 + x + 2 \Rightarrow 0 = x^2 + (1 - y)x + 2 + y = 0$$

$$\Rightarrow \Delta \geq 0 \Rightarrow (1 - y)^2 - 4(2 + y) \geq 0 \Rightarrow y^2 - 2y + 1 - 8 - 4y \geq 0$$

$$\Rightarrow y^2 - 2y - 7 \geq 0 \quad + \quad -1 - \quad - \quad + \Rightarrow y \in (-\infty, -1] \cup [7, +\infty)$$

$$x_5 = -\frac{b}{2a} = -\frac{1}{2} \rightarrow f\left(-\frac{1}{2}\right) = \frac{17}{4}$$

$$\text{الف) } y = 2x^2 - 5x + 1 \quad x_5 = \frac{5}{4} \Rightarrow y = 2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) + 1 = 1 \quad R_y = [1, +\infty)$$

$$b) y = \sqrt{-x^2 + 4x - 5} \quad x_5 = -\frac{b}{2a} = 2 \quad y_5 = -1 \quad y = \sqrt{-\infty, -1} \Rightarrow y = [0, 2] \checkmark$$

$$c) y = \sqrt{x^2 - 3x^2 + 4x + 1} \Rightarrow \sqrt{R} = [0, +\infty) = R_y$$

$$d) \log(x^2 - 4x^2 + 7x + 4) \Rightarrow \log(-3x^2 + 7x + 4) = (-\infty, +\infty) = IR$$

$$\text{اف) } y = \frac{3x + 1}{x^2 - 4} = \frac{3x + 1}{x^2 - 4} \quad R_y = IR - \left\{\frac{3}{4}\right\} \checkmark$$

$$\text{ب) } y = \sqrt{\frac{4x + 1}{x - 2}} \quad R_{\frac{4x+1}{x-2}} = IR - \{2\} \quad R_y = \sqrt{IR - \{2\}} = [0, +\infty) - \{2\}$$

$$2) y = \frac{x^2 + 4}{\sqrt{x^2 + 4}} \quad y = \frac{x^2 + 4}{\sqrt{x^2 + 4}} + \frac{1}{\sqrt{x^2 + 4}} = \frac{\sqrt{x^2 + 4} + 1}{\sqrt{x^2 + 4}} \quad R_y = \left[\sqrt{\frac{1}{10}} + \frac{1}{\sqrt{10}}, +\infty\right)$$

$$3) y = x^2 + \frac{1}{x^2 + 4} \Rightarrow y = x^2 + 4 + \frac{1}{x^2 + 4} - 4 \quad x = 0 \Rightarrow 4 + \frac{1}{4} = 4.25 = 17/4 \quad R_y = \left[\frac{17}{4}, +\infty\right) \checkmark$$

$$y = \frac{3x^2 + 4}{x^2 - 1} \quad yx^2 - y = 3x^2 + 4 \Rightarrow (y - 3)x^2 - y - 4 = 0$$

$$\Rightarrow \Delta \geq 0 \Rightarrow 0 - 4(y - 3)(-y - 4) \geq 0 \Rightarrow y \in (-\infty, -4] \cup [3, +\infty)$$

$$y = 3 \quad 3x^2 - 3 = 3x^2 + 4 \quad 3 \neq 4 \Rightarrow y \neq 3$$

$$y = \frac{3x^2 + 4}{x^2 - 1} \quad x = 0 \Rightarrow \frac{4}{-1} = -4 \quad y = \frac{3 \cdot \infty^2}{\infty^2 - 1} = 3$$

$$\Rightarrow y \in (-\infty, -4] \cup [3, +\infty)$$

$$y = \frac{2 \sin x - 1}{\sin x + 3} \Rightarrow \sin x = -\frac{3y + 1}{y - 2} = \frac{3y + 1}{2 - y}$$

$$-1 \leq \frac{3y + 1}{2 - y} \leq 1 \Rightarrow \frac{3y + 1}{2 - y} \leq 1 \quad \frac{3y + 1}{2 - y} \geq -1$$

$$\Rightarrow \frac{3y + 1}{2 - y} \leq 1 \Rightarrow \frac{3y + 1}{2 - y} - 1 \leq 0 \Rightarrow \frac{3y + 1 - 2 + y}{2 - y} \leq 0 \Rightarrow \frac{4y - 1}{2 - y} \leq 0$$

$$\Rightarrow y \in \left[\frac{1}{4}, 2\right)$$

$$y = \frac{2 \sin x - 1}{\sin x + 3} \quad y = \frac{2(1) - 1}{1 + 3} = \frac{1}{4} \quad y = \frac{2(-1) - 1}{-1 + 3} = -\frac{3}{2}$$

$$y \in \left[-\frac{3}{2}, \frac{1}{4}\right]$$

$$f(x) = \frac{x^2 - x + \frac{1}{2}}{x^2 - x + 1} \Rightarrow y$$

$$x^2 - x + 1 \neq 0 \quad \Delta < 0 \Rightarrow D_f = \mathbb{R}$$

$$\text{I } y \neq 1 \quad (y-1)^2 - 2(y-1)(y-\frac{1}{2}) \geq 0$$

$$y(x^2 - x + 1) = x^2 - x + \frac{1}{2}$$

$$(y-1)x^2 - x(y-1) + y - \frac{1}{2} = 0$$

$$y^2 - 2y + 1 - 2y^2 + 2y - 1 \geq 0$$

$$-y^2 + 1 \geq 0 \Rightarrow y^2 - 1 \leq 0$$

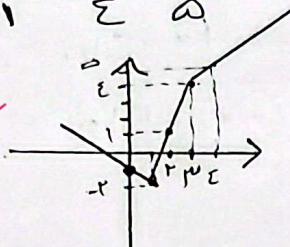
$$y = 1 \quad x^2 - x + 1 = x^2 - x + \frac{1}{2} \quad \text{GG II}$$

$$\text{I } y \in [0, 1)$$

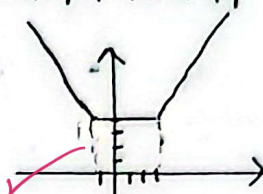
$$\text{a) } y = |x-2| - |x-3|$$

x	0	1	2	3	ε
y	-1	-2	1	ε	ω

$$R_f = [-2, +\infty)$$



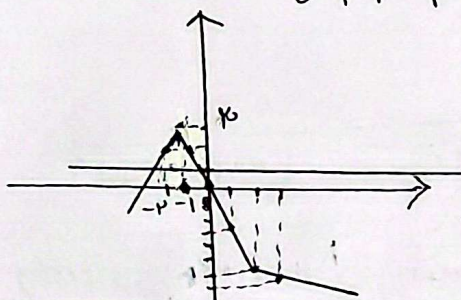
$$\text{b) } y = |x-3| + |x+1|$$



$$R_f = [\epsilon, +\infty)$$

$$|x-1| - |x+2| \leq 1$$

x	-2	-1	0	1	2
y	1	0	-1	-2	-1

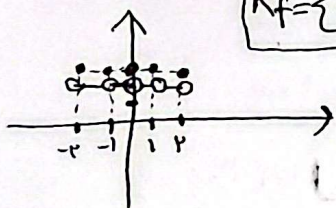


$$x \in (-3, -1] \cup [-\frac{1}{2}, +\infty)$$

$$\text{a) } y = [x] + [1-x] \quad [-2, 2]$$

$$y = [x] + [-x] + 1$$

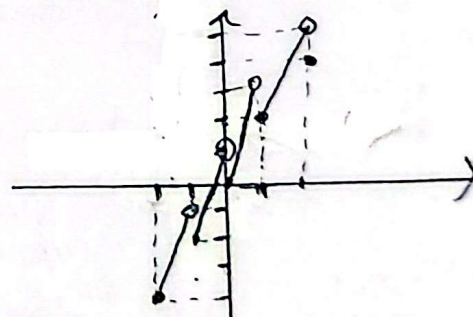
$$R_f = \{1, 0\}$$



$$\text{b) } y = 2x - [x] \quad [-2, 2]$$

$2x+1$	$-1 \leq x < 0$
$2x+1$	$-1 \leq x < 0$
$2x$	$0 \leq x < 1$
$2x-1$	$1 \leq x < 2$
ϵ	$x=2$

$$R_f = [-\epsilon, \omega)$$



$$\text{a) } y = \sin^2 x - \sin x + 1 \quad y = 1 - 1 + 1 = 1$$

$$\sin x = -1 \quad y = 1 + 1 + 1 = 3$$

$$\sin x = -\frac{1}{2} \quad y = \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}$$

$$R_f = [\frac{3}{4}, 3]$$

$$\text{b) } y = \sin^2 x + \sin^2 x + 1$$

$$\sin^2 x = 1 \quad y = 3$$

$$\sin^2 x = 0 \quad y = 1$$

$$R_f = [1, 3]$$

$$\text{GG } \sin^2 x = -\frac{1}{2}$$