

۱- $y(u-1) = u^2 \cdot u + 2 \Rightarrow yu - y = u^3 + 2u + 2 \Rightarrow u^3 + u(1-y) + 2 + y = 0 \Rightarrow u = \frac{-(1-y) \pm \sqrt{\Delta}}{2}$ (۲)
 $\Delta \geq 0 \Rightarrow (1-y)^2 - 4(1)(2+y) \geq 0 \Rightarrow 1 + y^2 - 4y - 8 - 4y \geq 0 \Rightarrow y^2 - 8y - 7 \geq 0$
 $(y-7)(y+1) \geq 0 \Rightarrow \frac{-1}{-1+1} \Rightarrow y \in (-\infty, -1] \cup [7, +\infty)$ ✓

۲- الف) $2u^2 - 5u + 1 = y \Rightarrow \begin{cases} u_{min} = \frac{-b}{2a} = \frac{-(-5)}{2(2)} = \frac{5}{4} \\ y_{min} = 2(\frac{5}{4})^2 - 5(\frac{5}{4}) + 1 = \frac{25}{8} - \frac{25}{4} + 1 = \frac{25}{8} - \frac{50}{8} + \frac{8}{8} = \frac{-27}{8} \Rightarrow [-\frac{27}{8}, +\infty) \end{cases}$ (۲)

۲- ب) $y = \sqrt{-u^2 + 4u - 5} \xrightarrow{\Delta \geq 0} \begin{cases} u_{max} = \frac{-b}{2a} = \frac{-4}{2(-1)} = 2 \\ y_{max} = - (2)^2 + 4(2) - 5 = -4 \end{cases} \Rightarrow R = \sqrt{(-\infty, 4]} = [0, 2]$ ✓

۲- ج) $y = \sqrt{u^2 - 4u + 5} \Rightarrow R_f = [0, +\infty)$ ✓

۲- د) $\log(u^2 - 4u + 5, \sqrt{u+1}) \Rightarrow R_g = (-\infty, +\infty)$ ✓

۳- الف) $y = \frac{3u+1}{u-4} \rightarrow \mathbb{R} - \{\frac{4}{3}\} \Rightarrow R_f = \mathbb{R} - \{\frac{4}{3}\}$ ✓

۳- ب) $y = \sqrt{\frac{4u+1}{u-2}} \rightarrow \sqrt{\mathbb{R} - \{\frac{2}{3}\}} \rightarrow R_g = [0, +\infty) - \{\frac{2}{3}\}$ ✓

۳- ج) $y = \frac{u^2+4}{\sqrt{u^2+2}} \cdot \frac{u^2+2}{\sqrt{u^2+2}} + \frac{1}{\sqrt{u^2+2}} \Rightarrow \sqrt{u^2+2} + \frac{1}{\sqrt{u^2+2}}$ $\xrightarrow{u=0}$ $\sqrt{2} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$ $\xrightarrow{\text{کثرین مقدار دارد}}$ $\Rightarrow [\frac{3}{\sqrt{2}}, +\infty)$ ✓

۳- د) $u^2 + 4 + \frac{1}{u^2+2} - 4 \xrightarrow{u=0} u^2 + 4 = 4 \Rightarrow [4, \frac{1}{4} - 4, +\infty) \Rightarrow [\frac{1}{4}, +\infty)$ ✓

۴- الف) $(\text{ریشه اول}) \rightarrow 3u^2 + 4 = yu^2 - y \Rightarrow u^2(3-y) = -y-4 \Rightarrow u^2 = \frac{-(y+4)}{3-y} \xrightarrow{u^2 \geq 0} \frac{-(y+4)}{3-y} \geq 0 \Rightarrow \frac{y+4}{3-y} \leq 0 \Rightarrow \frac{-4}{-1+1} \Rightarrow (-\infty, -4] \cup (3, +\infty)$ (۲)

۴- ب) $u=0 \rightarrow \frac{3(0)+4}{(0)^2-1} = -4 \quad u \rightarrow +\infty \rightarrow \frac{3(+\infty)+4}{(+\infty)^2-1} = 0$ $\Rightarrow (-\infty, -4] \cup (3, +\infty)$
 چون صحنه می تواند منفی شود پس برد خارج این عدد است. ✓

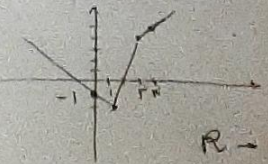
۵- الف) $(\text{ریشه اول}) \rightarrow y \sin u + 3y = 2 \sin u - 1 \Rightarrow \sin u(y-2) = -3y-1 \Rightarrow \sin u = \frac{-3y-1}{y-2}$
 $-1 \leq \frac{-3y-1}{y-2} \leq 1 \Rightarrow -1 \leq \frac{-3y-1}{y-2} \Rightarrow \frac{-3y-1-y+2}{y-2} \geq 0 \Rightarrow \frac{-4y+1}{y-2} \geq 0 \Rightarrow \frac{-4}{-1+1} \Rightarrow [-\frac{1}{4}, 2)$ (۲)

$\Rightarrow \frac{-3y-1}{y-2} \leq 1 \Rightarrow \frac{-3y-1-y+2}{y-2} \leq 0 \Rightarrow \frac{-4y+1}{y-2} \leq 0 \Rightarrow \frac{-4}{-1+1} \Rightarrow (-\infty, -\frac{1}{4}] \cup (2, +\infty)$ (II)

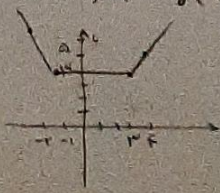
II $\Rightarrow [-\frac{1}{4}, \frac{1}{4}]$
 ۴- ب) $\sin u = -1 \rightarrow \frac{2(-1)-1}{(-1)+2} = \frac{-3}{1} = -3 \quad \sin u = 1 \rightarrow \frac{2(1)-1}{(1)+2} = \frac{1}{3}$
 صحنه می تواند منفی شود $\Rightarrow [-\frac{1}{4}, \frac{1}{4}]$ ✓

۶- دانه $\rightarrow \mathbb{R} - \{\text{صفر و کسری صفر}\} \Rightarrow u^2 \cdot u + 1 \neq 0 \Rightarrow \frac{-b \pm \sqrt{\Delta}}{2a} \neq 0 \Rightarrow \frac{-b \pm \sqrt{1-4(u)(u)}}{2} \neq 0 \Rightarrow \Delta \geq 0$
 صحنه همیشه ندارد $\Rightarrow R_g = \mathbb{R} \cdot \leftarrow$
 $y = \frac{u^2-u+1}{u^2-u+1} \rightarrow yu^2 - yu + y = u^2 - u + 1 \Rightarrow u = \frac{(y-1) \pm \sqrt{\Delta}}{2}$
 $\Rightarrow u^2(y-1) + u(-y+1) + y - \frac{1}{2} = 0 \xrightarrow{\Delta \geq 0} (-y+1)^2 - 4(y-1)(y-\frac{1}{2}) \geq 0$
 $y^2 - 2y + 1 - 4y^2 + 4y - 1 \geq 0 \Rightarrow -3y^2 + 2y \geq 0 \Rightarrow -3y(y-\frac{2}{3}) \geq 0 \Rightarrow \frac{1}{-1+1} \Rightarrow [0, \frac{2}{3}]$
 (۱) $R_f = \frac{2}{3}$ و اما از صفر کمتر است!
 (۱, ۲, ۳)

۷- الف) $f(1) = -2 \quad f(4) = 5 \quad f(3) = 4 \quad f(0) = -1$
 ب) $f(3) = 4 \quad f(2) = 5 \quad f(-1) = 4 \quad f(-2) = 6$

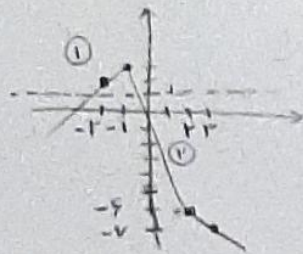


$R = [-2, +\infty)$



$R = [4, +\infty)$

۸- $f(x) = -5$ $f(x) = -7$
 $f(-1) = 3$ $f(-2) = 2$

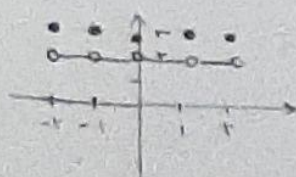


حیث حاصل باید کوچکتر از ۱ باشد طبق نمودار قبل بر دو خط اد ۲ را با ۱ تعیین می کنیم و طبق آن بازه مطلوب را می نویسیم.

۱ خط $\rightarrow -2x - 2 - (-2x + 2) = 1 \Rightarrow x + 2 = 1 \Rightarrow x = -1$
 ۲ خط $\rightarrow x - 2 - (-2x + 2) = 1 \Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$
 $D_f \rightarrow (-\infty, -1] \cup [\frac{1}{3}, +\infty)$

۹- الف) $[u] + [3-u] = [u] + [-u] + 3$

$\rightarrow [u] + [-u] = 0 \quad u \in \mathbb{Z} \rightarrow [u] + [-u] + 3 = 3$
 $\rightarrow [u] + [-u] = -1 \quad u \notin \mathbb{Z} \rightarrow [u] + [-u] + 3 = 2$

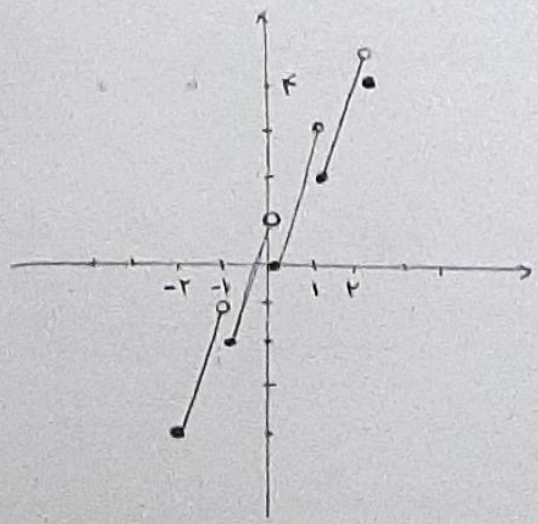


$R_f = \{2, 3\}$

۹- ب) $3u - [u]$

$R_f = [-2, 0]$

$-2 \leq u < -1 \rightarrow 3u - (-2) = 3u + 2$
 $[u] = -2$
 $-1 \leq u < 0 \rightarrow 3u - (-1) = 3u + 1$
 $[u] = -1$
 $0 \leq u < 1 \rightarrow 3u - (0) = 3u$
 $[u] = 0$
 $1 \leq u < 2 \rightarrow 3u - (1) = 3u - 1$
 $[u] = 1$



۱۰ الف) $\sin^2 u - \sin u + 1$

$\rightarrow \sin u = -1 \rightarrow y = 3$
 $\rightarrow \sin u = 1 \rightarrow y = 1$
 $\rightarrow \sin u = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2} \rightarrow y = \frac{3}{2}$

$R_f = [\frac{3}{2}, 3]$

ب) $y = \sin^2 u + \sin^2 u + 1$

$\rightarrow \sin^2 u = 0 \rightarrow y = 1$
 $\rightarrow \sin^2 u = 1 \rightarrow y = 3$
 $\rightarrow \sin^2 u = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2} \notin [0, 1]$

$R_f = [1, 3]$