

الف) $y = \frac{x^2 + x + 2}{x - 1} \Rightarrow y(x - 1) = x^2 + x + 2 \Rightarrow x^2 + (1 - y)x + 2 + y = 0$ $\xrightarrow{\text{از این معادله درجه دوم در x}} \Delta \geq 0 \Rightarrow (1 - y)^2 - 4(2 + y) \geq 0$
 $= y^2 - 2y - 7 \geq 0$ $\xrightarrow{\text{نمودار}} \begin{array}{c} \text{+} \\ \text{0} \\ \text{-} \end{array} \begin{array}{c} \text{+} \\ \text{0} \\ \text{-} \end{array}$
 $\Rightarrow R_f: (-\infty, -1] \cup [7, +\infty)$

الف) $y = 2x^2 - 5x + 1 \rightarrow y_{\min} = -\frac{\Delta}{4a} = -\frac{(25 - 8)}{8} = -\frac{17}{8} \Rightarrow R_f: [-\frac{17}{8}, +\infty)$
 ب) $y = \sqrt{-x^2 + 2x - 5} \rightarrow y_{\max} = \frac{\Delta}{4a} = -\frac{(4 - 20)}{-4} = 4 \rightarrow R_f: (-\infty, 4]$
 ج) $y = \sqrt{x^2 - 2x^2 + 4x + 1} \rightarrow$ $\xrightarrow{\text{این تابع درجه دوم در x است}} R_f: [0, +\infty)$
 د) $y = \log(x^2 - 2x^2 + 4x + 1) \rightarrow$ $\xrightarrow{\text{این تابع درجه دوم در x است}} R_f: (-\infty, +\infty)$

الف) $y = \frac{x^2 + 1}{x - 2} \Rightarrow (x - 2)y = x^2 + 1 \Rightarrow x^2 - 2yx + 2y - 1 = 0$ $\xrightarrow{\Delta \geq 0} R_f: \{R - \{\frac{1}{2}\}, R - \{\frac{5}{2}\}\}$
 ب) $y = \sqrt{\frac{x^2 + 1}{x - 2}} \Rightarrow (-2)(x) \neq 2 \rightarrow R_f: [0, +\infty) - \{\sqrt{\frac{1}{2}}\} \cup [2, +\infty) - \{\frac{5}{2}\}$
 ج) $y = \frac{x^2 + 4}{\sqrt{x^2 + 3}} \Rightarrow \sqrt{x^2 + 3} \cdot y = x^2 + 4 \Rightarrow R_f: [2, +\infty)$
 د) $y = x^2 + \frac{1}{x^2 + 4} \Rightarrow x^2 + 4 + \frac{1}{x^2 + 4} \geq 4 \Rightarrow R_f: [\frac{1}{4}, +\infty)$

الف) $y = \frac{x^2 + 4}{x^2 - 1} \Rightarrow x^2 = \frac{y + 4}{y - 3} \Rightarrow \frac{y + 4}{y - 3} \geq 0 \Rightarrow R_f: (-\infty, -4] \cup (3, +\infty)$
 ب) $\min x^2 = 0 \rightarrow \frac{4}{(-1) - 1} = -2 \Rightarrow y = 2$
 $\max x^2 = +\infty \rightarrow y = 3$
 $\Rightarrow R_f: (-\infty, 2] \cup (3, +\infty)$

الف) $y = \frac{2 \sin x - 1}{\sin x + 3} \Rightarrow \sin x = \frac{2y + 1}{y - 3} = \frac{2y + 1}{y - 3}$ $\xrightarrow{\text{نمودار}} \begin{array}{c} \text{+} \\ \text{0} \\ \text{-} \end{array} \begin{array}{c} \text{+} \\ \text{0} \\ \text{-} \end{array}$
 ب) $\sin x_{\min} = -1 \rightarrow y = \frac{3}{1} = 3$
 $\sin x_{\max} = 1 \rightarrow y = \frac{1}{5}$
 $\Rightarrow R_f: [\frac{1}{5}, 3]$

$$P(n) = \frac{x^2 - n + \frac{1}{x}}{x^2 - n + 1}$$

$$D_f: x^2 - n + 1 \neq 0 \xrightarrow{\Delta \neq 0} \text{دو ریشه حقیقی} \rightarrow D_f: \mathbb{R}$$

$$R_f: \frac{x^2 - n + \frac{1}{x}}{x^2 - n + 1} = 1 - \frac{\frac{1}{x}}{x^2 - n + 1}$$

برای یافتن دامنه: $x^2 - n + 1 \rightarrow y_{\min} = \frac{\Delta}{4a} = \frac{-(1-4)}{4} = \frac{3}{4} \rightarrow R_f: [\frac{3}{4}, +\infty) \Rightarrow \frac{1}{x} \{ x^2 - n + 1 \} < +\infty \Rightarrow \frac{1}{x} \frac{1}{x^2 - n + 1} > \frac{1}{x^2}$

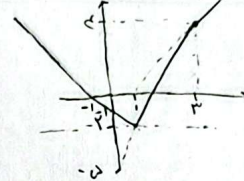
$\Rightarrow 1 > \frac{1}{x^2} > 0 \Rightarrow -1 < \frac{1}{x} < 0 \Rightarrow 0 < 1 - \frac{1}{x} < 1 \rightarrow R_f: [0, 1)$

الف) $|x_n - 2| + |x - 3| = 7$

حالت‌های مختلف:

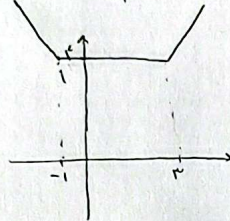
$$\begin{array}{c|c|c|c} \text{حالت 1} & \text{حالت 2} & \text{حالت 3} & \text{حالت 4} \\ \hline x \geq 3 & 2 \leq x < 3 & x < 2 & x < 2 \end{array}$$

$$\begin{array}{c|c|c|c} \hline (x-2) - (x-3) & (x-2) - (3-x) & (2-x) - (3-x) & (2-x) - (-x-3) \\ \hline -x+5 & 2x-5 & -1 & -2x-1 \end{array}$$



$$R_f: [-2, +\infty)$$

ب) $|x-2| + |x+1| = 7$



$$R_f: [2, +\infty)$$

$|x-2| - |x+2| \leq 1$

حالت‌های مختلف: $x \geq -1$ و $x < -1$ $\Rightarrow x \leq -3$

حالت‌های مختلف:

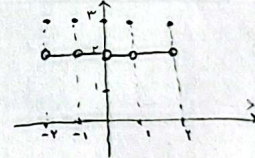
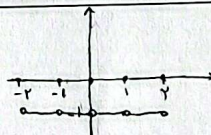
$$\begin{array}{c|c|c|c} \text{حالت 1} & \text{حالت 2} & \text{حالت 3} & \text{حالت 4} \\ \hline x \geq 2 & -1 < x < 2 & x < -1 & x < -2 \end{array}$$

$$\begin{array}{c|c|c|c} \hline (x-2) - (x+2) & (x-2) - (-x-2) & (-x-2) - (x+2) & (-x-2) - (-x-2) \\ \hline -4 & 2x & -2x-4 & 0 \end{array}$$

نتیجه: $x \leq -3$ یا $x \geq 2$

دامنه: $(-\infty, -3] \cup [2, +\infty)$

الف) $y = [x] + [x-n] = [x] + [-n] + 3$

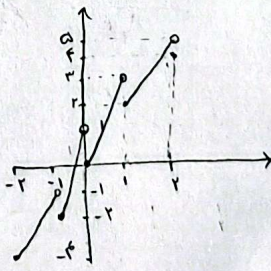


ب) $3n - [x]$

برای $x \in [-1, 1]$:

- $x \in [-1, 0)$: $[x] = -1 \Rightarrow 3n - (-1) = 3n + 1$
- $x \in [0, 1)$: $[x] = 0 \Rightarrow 3n - 0 = 3n$
- $x \in [1, 2)$: $[x] = 1 \Rightarrow 3n - 1$
- $x = 2$: $[x] = 2 \Rightarrow 3n - 2$

$[x] + [-n]$



الف) $y = \sin^2 n - \sin n + 1$

دامنه: $y \in [1, 3]$

ب) $y = \sin^2 n + \sin^2 n + 1$

دامنه: $y \in [1, 3]$