

(۱) (۲)
 $y^2 - y = x^2 + x + 1$ $x^2 + (1-y)x + 1+y = 0$ $\Delta \geq 0 \rightarrow (1-y)^2 - 4(1+y) \geq 0$
 $y^2 - 2y + 1 - 4 - 4y \geq 0$ $y^2 - 6y - 3 \geq 0$ $\frac{-1 \pm \sqrt{1+12}}{2}$ $R_f = (-\infty -1] \cup [3 + \infty)$ ✓

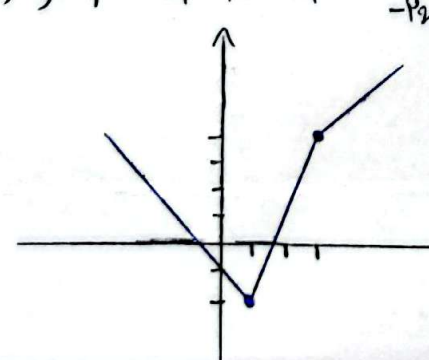
(۲) (۲)
 الف) $y = 2x^2 - 5x + 1$ $\text{طول رأس} = \frac{5}{2} \rightarrow \text{عرض رأس} = \frac{-17}{2}$ $R_f = [-\frac{17}{2} + \infty)$ ✓
 ب) $y = \sqrt{x^2 + 9x - 4}$ $\text{طول رأس} = 3 \rightarrow \text{عرض رأس} = 4$ $R_f = [0, 2]$ ✓
 ج) $y = \sqrt{x^2 - 2x^2 + 5x + 1}$ $\text{بزرگ ترین توان عبارت زیر رادیکال فرد است} \rightarrow R_f = \sqrt{R} = [0 + \infty)$ ✓
 د) $y = \log(x^3 - 6x^2 + 7x + 4)$ $R_f = \mathbb{R}$ ✓

(۳) (۱, ۱/۵)
 الف) $y = \frac{3x+1}{2x-4}$ $\text{هموگرافیک} \rightarrow R_f = \mathbb{R} - \{\frac{3}{2}\}$ ✓
 ب) $y = \sqrt{\frac{5x+1}{x-2}}$ $R_f = [0 + \infty) - \{\frac{1}{2}\}$ ✓ $x < 0 \rightarrow \min y = \frac{4\sqrt{5}}{5} \rightarrow R_f = [\frac{4\sqrt{5}}{5} + \infty)$
 ج) $y = \frac{x^2+4}{\sqrt{x^2+3}} = \frac{x^2+3+1}{\sqrt{x^2+3}} = \sqrt{x^2+3} + \frac{1}{\sqrt{x^2+3}}$ $\frac{\sqrt{x^2+3}}{1} > 0 \rightarrow R_f = [2 + \infty)$ ✓
 د) $y = x^2 + \frac{1}{x^2+4} = x^2 + 4 + \frac{1}{x^2+4} - 4$ $R_f = [0, 1/4 + \infty)$

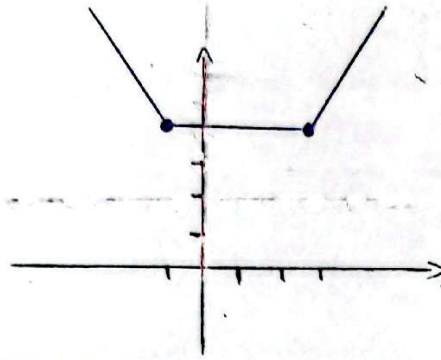
(۴) (۲)
 الف) $y = \frac{3x^2+4}{x^2-1} \rightarrow x^2 = \frac{-y-4}{y-3} = \frac{y+4}{y-3}$ $x^2 \geq 0 \rightarrow \frac{y+4}{y-3} \geq 0$ $\frac{-4 \pm 3}{2} \rightarrow R_f = (-\infty -4] \cup (3 + \infty)$
 ب) $x^2 = 0 \rightarrow y = -4$ $x^2 = +\infty \rightarrow y = 3$ $\text{منفرد ریشه دارد} \rightarrow R_f = (-\infty -4] \cup (3 + \infty)$

(۵) (۲)
 الف) $y = \frac{1 \sin x - 1}{\sin x + 3} \rightarrow \sin x = \frac{3y+1}{y-2} = \frac{-3y-1}{y-2}$ $-1 \leq \sin x \leq 1 \rightarrow -1 \leq \frac{-3y-1}{y-2} \leq 1$
 $\frac{-3y-1+y-2}{y-2} \geq 0$ $\frac{-2y-3}{y-2} \geq 0$ $\frac{-\frac{3}{2} \pm 2}{-1 \pm 1} \text{ I}$ $\frac{-3y-1-y+2}{y-2} \leq 0$ $\frac{-4y+1}{y-2} \leq 0$ $\frac{\frac{1}{4} \pm 2}{-1 \pm 1} \text{ II}$
 $I \cap II \rightarrow R_f = [-\frac{3}{2}, \frac{1}{4}]$
 ب) $\sin x = -1 \rightarrow y = -\frac{3}{2}$ $\sin x = 1 \rightarrow y = \frac{1}{4}$ $\text{منفرد ریشه ندارد} \rightarrow R_f = [-\frac{3}{2}, \frac{1}{4}]$

(۶) (۲)
 $4x^2 - yx + y = x^2 - x + \frac{1}{4}$ $(y-1)x^2 - (y-1)x + y - \frac{1}{4} = 0$ $\Delta \geq 0$ $y \neq 1$ $(y-1)^2 - 4(y - \frac{1}{4})(y-1) \geq 0$ $(y-1)(y-1-4y+1) \geq 0$ $\frac{0 \pm 1}{-1 \pm 1} \rightarrow R_f = [0, 1]$ ✓
 $x^2 - x + 1 \xrightarrow[\Delta > 0]{\Delta \leq 0}$ همواره مثبت $R_f = \mathbb{R}$

(۷) (۲)
 الف) $y = |2x-2| - |x-3|$

 $\begin{array}{c|c|c} 1 & 3 & \\ \hline -2x+2+x-3 & 2x-2+x-3 & 2x-2-x+3 \\ \hline -x-1 & 3x-5 & x+1 \end{array}$
 $R_f = [-2 + \infty)$ ✓

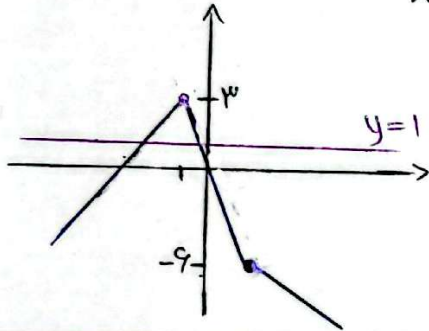
ب) $y = |x-3| + |x+1|$



$R_f = [-\infty, +\infty)$

(۷)

$y = |x-2| - |2x+2| \leq 1$



$$\begin{array}{c|c|c|c|c} & -1 & & 2 & \\ \hline -x+2+2x+2 & -x+2-2x-2 & x-2-2x-2 & & \\ x+4 & -3x & -x-4 & & \end{array}$$

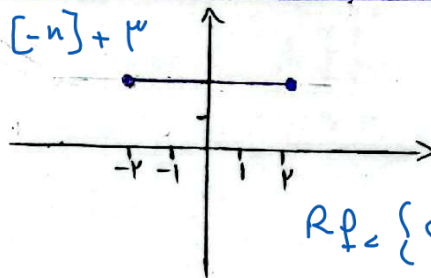
$$\begin{cases} x < -1 \rightarrow x+4 \leq 1 & x \leq -3 \\ -1 \leq x \leq 2 \rightarrow -3x \leq 1 & x \geq -\frac{1}{3} \\ x \geq 2 \rightarrow \text{طبق شکل ریشه ندارد} \end{cases}$$

$D_f = (-\infty, -3] \cup [-\frac{1}{3}, +\infty)$

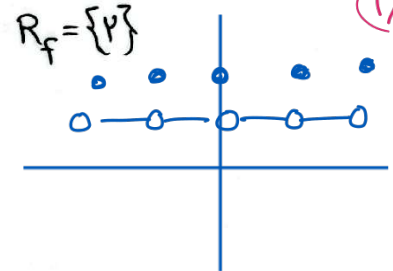
(۸)

الف) $y = [x] + [x-1] \rightarrow [x] + [-x] + 1$

$$\begin{cases} 1 & -2 \leq x < -1 \\ 1 & -1 \leq x < 0 \\ 1 & 0 \leq x < 1 \\ 1 & 1 \leq x < 2 \\ 1 & x = 2 \end{cases}$$



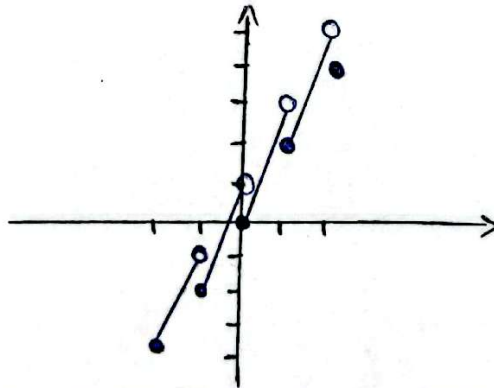
$R_f = \{1\}$



(۹)

ب) $y = 3x - [x]$

$$\begin{cases} 3x+2 & -2 \leq x < -1 \\ 3x+1 & -1 \leq x < 0 \\ 3x & 0 \leq x < 1 \\ 3x-1 & 1 \leq x < 2 \\ 4 & x = 2 \end{cases}$$



$R_f = [-\infty, \infty)$

الف) $\sin^2 x - \sin x + 1$

$$\begin{cases} \sin x = -1 \rightarrow y = 3 \\ \sin x = 1 \rightarrow y = 1 \\ \sin x = \frac{-b}{1a} = \frac{1}{1} \rightarrow y = \frac{3}{1} \end{cases}$$

$R_f = [\frac{3}{1}, 3]$

(۱۰)

ب) $\sin^2 x + \sin^2 x + 1$

$$\begin{cases} \sin^2 x = 0 \rightarrow y = 1 \\ \sin^2 x = 1 \rightarrow y = 3 \\ \sin^2 x = \frac{-b}{1a} = \frac{-1}{1} \end{cases}$$

$R_f = [1, 3]$