

(1) $y^2 - y = x^2 + x + 1$ $x^2 + (1-y)x + 1+y = 0$ $\Delta \geq 0 \rightarrow (1-y)^2 - 4(1+y) \geq 0$
 $y^2 - 2y + 1 - 4 - 4y \geq 0$ $y^2 - 6y - 3 \geq 0$ $\frac{-1 \pm \sqrt{1+12}}{2} R_f = (-\infty -1] \cup [3 + \infty)$

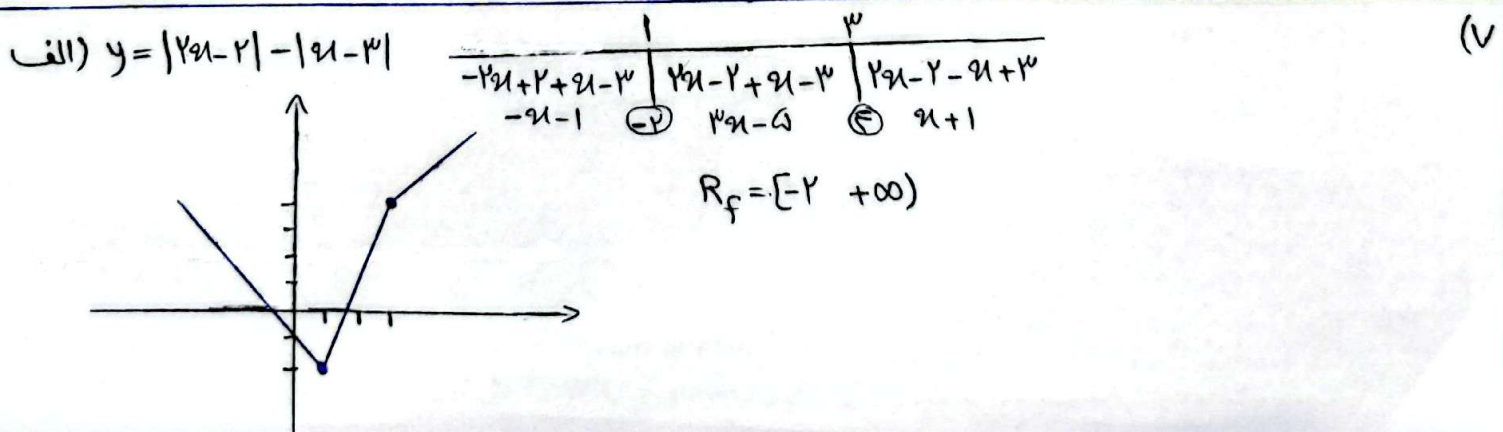
(2) الف) $y = 2x^2 - 5x + 1$ طول رأس = $\frac{5}{4} \rightarrow$ عرض رأس = $-\frac{17}{8}$ $R_f = [-\frac{17}{8} + \infty)$
 ب) $y = \sqrt{-x^2 + 6x - 5}$ طول رأس = 3 $\rightarrow$ عرض رأس = 4 $R_f = [0, 2]$
 ج) $y = \sqrt{x^2 - 2x^2 + 5x + 1}$ بزرگترین توان عبارت زیر رادیکال فرد است $\rightarrow R_f = \sqrt{1R} = [0 + \infty)$
 د) $y = \log(x^3 - 6x^2 + 7x + 4)$ $R_f = 1R$

(3) الف) $y = \frac{3x+1}{2x-4}$ همگرافیک $\rightarrow R_f = 1R - \{ \frac{3}{2} \}$
 ب) $y = \sqrt{\frac{5x+1}{x-2}}$ $R_f = [0 + \infty) - \{2\}$
 ج) $y = \frac{x^2+4}{\sqrt{x^2+3}} = \frac{x^2+3+1}{\sqrt{x^2+3}} = \sqrt{x^2+3} + \frac{1}{\sqrt{x^2+3}}$ $\sqrt{x^2+3} > 0 \rightarrow R_f = [2 + \infty)$
 د) $y = x^2 + \frac{1}{x^2+4} = x^2 + 4 + \frac{1}{x^2+4} - 4$ $R_f = [0, 12.5 + \infty)$

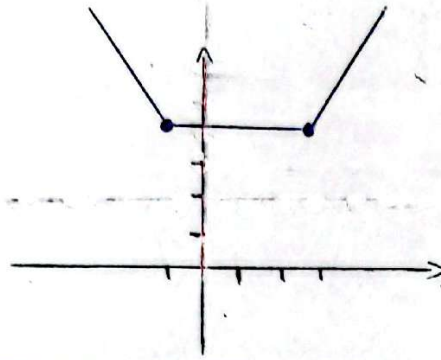
(4) الف) (روش اصلی): $y = \frac{3x^2+4}{x^2-1} \rightarrow x^2 = \frac{-y-4}{y-3} = \frac{y+4}{y-3}$ $x^2 \geq 0 \rightarrow \frac{y+4}{y-3} \geq 0$ $\frac{-4 \pm \sqrt{16+9}}{2} R_f = (-\infty -4] \cup (3 + \infty)$
 ب) (روش دوم): $x^2 = 0 \rightarrow y = -4$ $x^2 = +\infty \rightarrow y = 3$ $\rightarrow R_f = (-\infty -4] \cup (3 + \infty)$ مخرج ریشه دار

(5) الف) (روش اصلی): $y = \frac{1 \sin x - 1}{\sin x + 3} \rightarrow \sin x = \frac{3y+1}{y-2} = \frac{-3y-1}{y-2}$ $-1 \leq \sin x \leq 1 \rightarrow -1 \leq \frac{-3y-1}{y-2} \leq 1$
 $\frac{-3y-1+y-2}{y-2} \geq 0$ $\frac{-2y-3}{y-2} \geq 0$ $\frac{-\frac{3}{2} \pm \sqrt{\frac{9}{4}+9}}{2} \text{ I}$ $\frac{-3y-1-y+2}{y-2} \leq 0$ $\frac{-4y+1}{y-2} \leq 0$ $\frac{\frac{1}{4} \pm \sqrt{\frac{1}{16}+9}}{2} \text{ II}$
 $I \cap II \rightarrow R_f = [-\frac{3}{2}, \frac{1}{4}]$
 ب) (روش دوم): $\sin x = -1 \rightarrow y = -\frac{3}{2}$ $\sin x = 1 \rightarrow y = \frac{1}{4}$ $\rightarrow R_f = [-\frac{3}{2}, \frac{1}{4}]$ مخرج ریشه ندارد

(6) $4x^2 - yx + y = x^2 - x + \frac{1}{4}$ $(y-1)x^2 - (y-1)x + y - \frac{1}{4} = 0$ $\Delta \geq 0$ $y \neq 1$ نمی تواند خط باشد پس
 $(y-1)^2 - 4(y-\frac{1}{4})(y-1) \geq 0$ $(y-1)(y-1-4y+1) \geq 0$ $\frac{0 \pm \sqrt{0+16}}{2} \rightarrow R_f = [0, 1]$
 $x^2 - x + 1 \frac{\Delta \leq 0}{\Delta > 0} \rightarrow$ همواره مثبت $R_f = 1R$



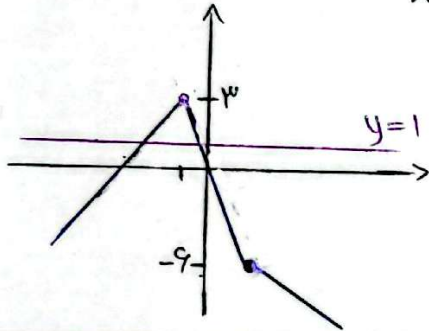
ب) $y = |x-3| + |x+1|$



$R_f = [-\infty, +\infty)$

(۷)

$y = |x-2| - |2x+2| \leq 1$



$$\begin{array}{c|c|c|c|c} & -1 & & 2 & \\ \hline -x+2+2x+2 & -x+2-2x-2 & x-2-2x-2 & & \\ x+4 & -3x & -x-4 & & \end{array}$$

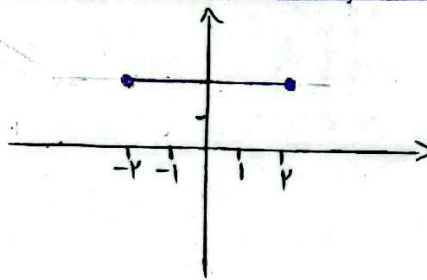
$$\begin{cases} x < -1 \rightarrow x+4 \leq 1 & x \leq -3 \\ -1 \leq x < 2 \rightarrow -3x \leq 1 & x \geq -\frac{1}{3} \\ x \geq 2 \rightarrow \text{طبق شکل ریشه ندارد} \end{cases}$$

$D_f = (-\infty, -3] \cup [-\frac{1}{3}, +\infty)$

(۸)

الف) $y = [x] + [3-x]$

$$\begin{cases} 2 & -2 \leq x < -1 \\ 2 & -1 \leq x < 0 \\ 2 & 0 \leq x < 1 \\ 2 & 1 \leq x < 2 \\ 2 & x = 2 \end{cases}$$

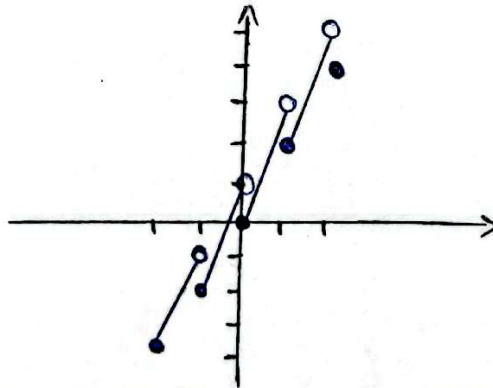


$R_f = \{2\}$

(۹)

ب) $y = 3x - [x]$

$$\begin{cases} 3x+2 & -2 \leq x < -1 \\ 3x+1 & -1 \leq x < 0 \\ 3x & 0 \leq x < 1 \\ 3x-1 & 1 \leq x < 2 \\ 4 & x = 2 \end{cases}$$



$R_f = [-4, \infty)$

الف) $\sin^2 x - \sin x + 1$

$$\begin{cases} \sin x = -1 \rightarrow y = 3 \\ \sin x = 1 \rightarrow y = 1 \\ \sin x = \frac{-b}{2a} = \frac{1}{2} \rightarrow y = \frac{3}{4} \end{cases}$$

$R_f = [\frac{3}{4}, 3]$

(۱۰)

ب) $\sin^2 x + \sin^2 x + 1$

$$\begin{cases} \sin^2 x = 0 \rightarrow y = 1 \\ \sin^2 x = 1 \rightarrow y = 3 \\ \sin^2 x = \frac{-b}{2a} = \frac{-1}{2} \text{ غوغ} \end{cases}$$

$R_f = [1, 3]$