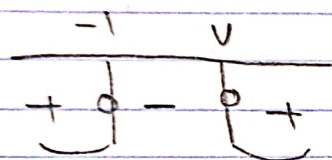


مهمانخانه / تکلیف شماره ۳

$$y = \frac{x^2 + 9x + 1}{x - 1} \Rightarrow yx - y = x^2 + 9x + 1 \rightarrow x^2 + (1 - y)x + 1 + y = 0 \quad (1)$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} \xrightarrow{\Delta \geq 0} b^2 - 4ac \geq 0 \rightarrow (1 - y)^2 - 4(1 + y) \geq 0 \rightarrow$$

$$1 + y^2 - 2y - 4 - 4y \geq 0 \rightarrow y^2 - 6y - 3 \geq 0 \xrightarrow{a + c = b} y = -1, y = 7$$



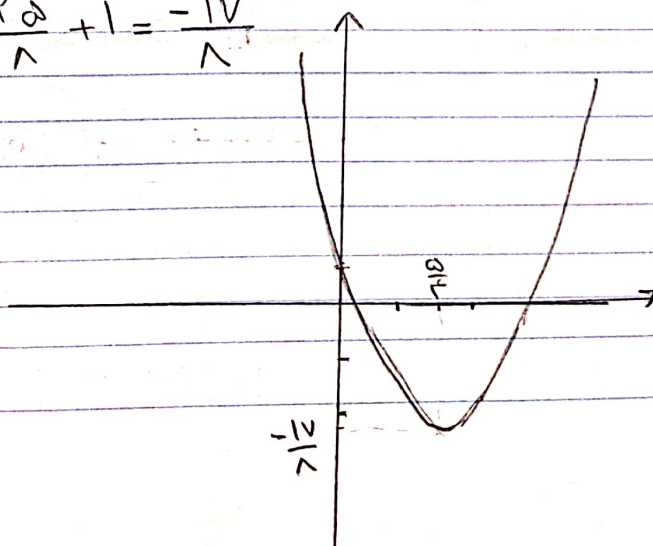
$$R_f = (-\infty, -1] \cup [7, +\infty)$$

الف) $y = 2x^2 - \omega x + 1$

$$x = \frac{-b}{2a} = \frac{\omega}{4}$$

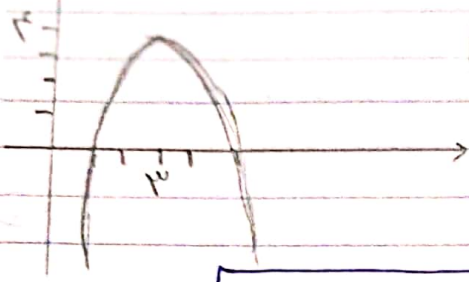
$$y = 2 \times \frac{\omega}{4} - \omega \times \frac{\omega}{4} + 1 = \frac{-\omega}{2} + 1 = \frac{-\omega}{2} + 1$$

$$R_f = \left[-\frac{\omega}{2}, +\infty\right)$$



ترس از خدا راس حکمت است. حضرت مسیح (ع)

ب) $y = \sqrt{-9x^2 + 49x - 5} \rightarrow x = \frac{-b}{2a} = \frac{-9}{-18} = \frac{1}{2} \quad y = -9 + 18 - 5 = 4$



$R_f = (-\infty, \sqrt{4}] \rightarrow [0, 2]$

ج) $y = \sqrt{9x^2 - 39x + 49x + 1}$ بزرگترین توان این عبارت فرد است پس $R_f = \mathbb{R}$

$R_f = \sqrt{\mathbb{R}} = [0, +\infty)$

د) $\log(9x^2 - 39x + 49x + 1)$ از $\log$ می‌گیریم $R_f = (-\infty, +\infty) = \mathbb{R}$
بد R دارد

(۳)

الف) $y = \frac{4x+1}{2x-4} \Rightarrow x = -\frac{-2y-1}{2y-3} = \frac{2y+1}{2y-3}$

$2y-3 \neq 0 \rightarrow y \neq \frac{3}{2} \quad R_f = \mathbb{R} - \{\frac{3}{2}\}$

ب) $y = \sqrt{\frac{4x+1}{2x-2}}$ توان ۲، $y^2 = \frac{4x+1}{2x-2} \rightarrow x = -\frac{-2y^2-1}{2y^2-4} = \frac{2y^2+1}{2y^2-4}$

$2y^2-4 \neq 0 \rightarrow y^2 \neq 2 \rightarrow y \neq \sqrt{2}, -\sqrt{2} \quad , y \geq 0$

$R_f = [0, +\infty) - \{\sqrt{2}\}$

ج) $y = \frac{x^2+4}{\sqrt{x^2+3}} = \frac{x^2+3+1}{\sqrt{x^2+3}} = \sqrt{x^2+3} + \frac{1}{\sqrt{x^2+3}}$

لکه کمترین مقدار $x=0$

$y = \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4\sqrt{3}}{3} \rightarrow R_f = [\frac{4\sqrt{3}}{3}, +\infty)$

$$5) y = x^2 + \frac{1}{x^2 + 4} + 4 - 4 = x^2 + 4 + \frac{1}{x^2 + 4} - 4 \xrightarrow{x=0} y = \frac{1}{4}$$

$\frac{1}{4} \leftarrow 0 = x \text{ مقدار } x \text{ را در } \frac{1}{x^2 + 4} \text{ قرار دهیم}$

$$R_f = [\frac{1}{4}, +\infty)$$

$$y = \frac{3x^2 + 4}{x^2 - 1}$$

$$\text{روش اول: } x^2 = -\frac{-y-4}{y-3} \rightarrow x = \pm \sqrt{\frac{y+4}{y-3}} \rightarrow \frac{y+4}{y-3} \geq 0 \rightarrow \frac{-4}{+} \frac{3}{-}$$

$$R_f = (-\infty, -4] \cup (3, +\infty)$$

$$\text{روش دوم: } y = \frac{3x^2 + 4}{x^2 - 1} \begin{cases} \min \rightarrow x^2 = 0 \rightarrow y = -4 \\ \max \rightarrow x^2 = +\infty \rightarrow y = \frac{3(\infty) + 4}{(\infty) - 1} \xrightarrow{\text{فاصله } -1, 4} \frac{3(\infty)}{\infty} = 3 \end{cases}$$

$$\text{مخرج } x=0, x=1, x=-1 \text{ را در } R_f = (-\infty, -4] \cup (3, +\infty)$$

$$y = \frac{2 \sin x - 1}{\sin x + 2}$$

$$\text{روش اول: } \sin x = -\frac{2y+1}{y-2} \rightarrow \sin x = \frac{2y+1}{2-y} \rightarrow -1 \leq \frac{2y+1}{2-y} \leq 1$$

$$\text{I) } \frac{2y+1}{2-y} \geq -1 \rightarrow \frac{2y+1}{2-y} \geq 0 \rightarrow \frac{-\frac{1}{2}}{-} \frac{2}{+} \left. \begin{array}{l} \text{II) } \frac{2y+1}{2-y} \leq 1 \rightarrow \frac{2y-1}{2-y} \leq 0 \rightarrow \frac{\frac{1}{2}}{-} \frac{2}{+} \end{array} \right\} \rightarrow R_f = [-\frac{1}{2}, \frac{1}{2}]$$

$$\text{min} \rightarrow \sin x = -1 \rightarrow y = -\frac{3}{F}$$

$$\text{max} \rightarrow \sin x = 1 \rightarrow y = \frac{1}{F}$$

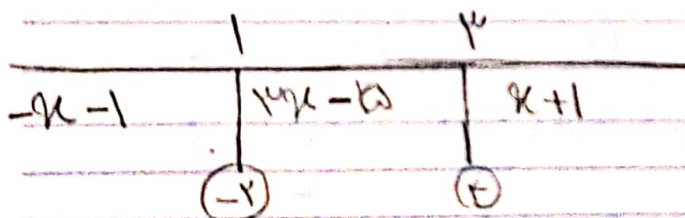
$$\sin x + \frac{3}{F} \rightarrow \text{range} \rightarrow R_f = \left[-\frac{3}{F}, \frac{1}{F}\right]$$

$$f(x) = \frac{x^2 - x + \frac{1}{F}}{x^2 - x + 1} \rightarrow x^2 - x + 1 \neq 0 \rightarrow \Delta < 0 \rightarrow \frac{0}{0} \rightarrow D_f = \mathbb{R}$$

$$y = \frac{(x - \frac{1}{2})^2}{(x - \frac{1}{2})^2 + \frac{3}{4}} \rightarrow (x - \frac{1}{2})^2 = -\frac{\frac{3}{4}y}{y-1} = \frac{\frac{3}{4}y}{1-y} \geq 0 \rightarrow \frac{0}{1}$$

$$R_f = [0, 1)$$

$$y = \frac{|2x-2| - |x-4|}{|2x-2| + |x-4|}$$



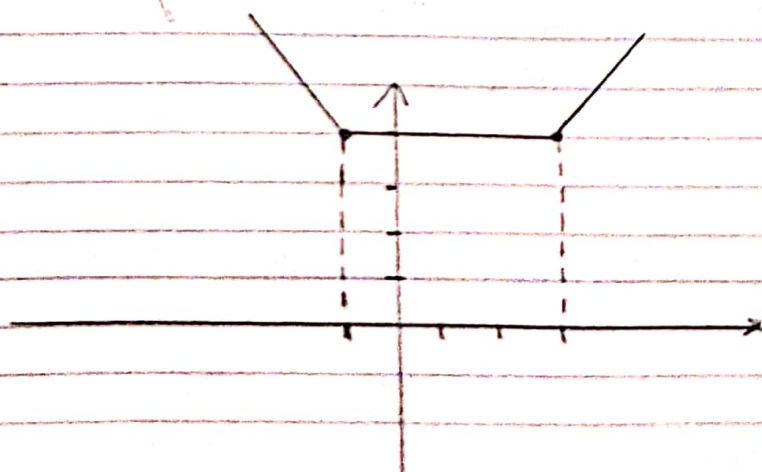
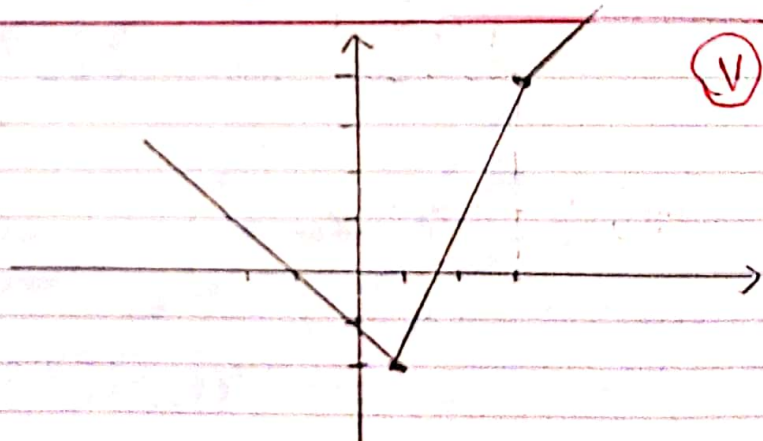
$$R_f = [-2, +\infty)$$

$$y = \frac{|x-3| + |x+1|}{|x-3| + |x+1|}$$

$$3 - (-1) = 4 \rightarrow \text{range}$$

له باسبب

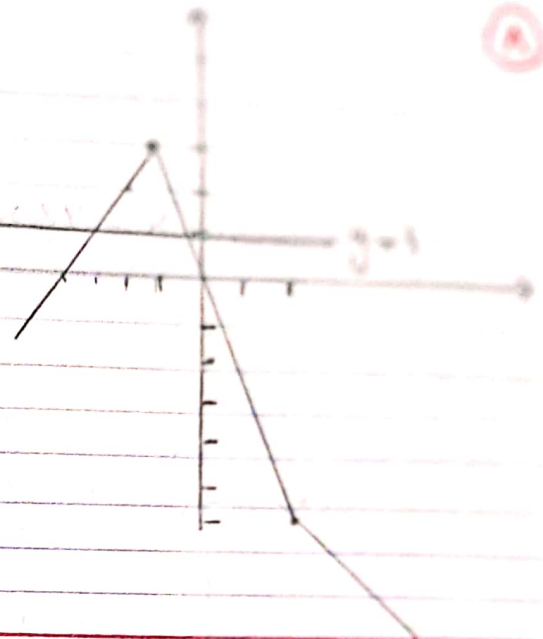
$$R_f = [4, +\infty)$$



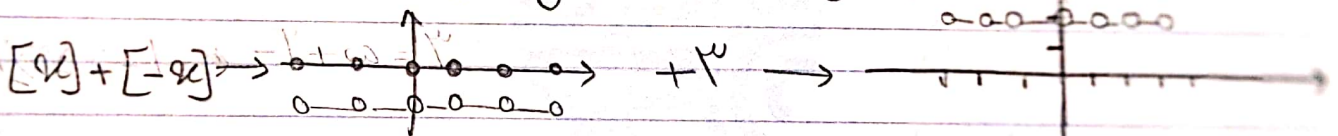
$$|x-1| - |x+1| \leq 1$$

$$\begin{array}{ccc} x+1=1 & -3x \leq 1 & -x-1 < 1 \\ x=0 & x \geq -\frac{1}{3} & x > -2 \end{array}$$

$$D_f = \mathbb{R} - (-3, -\frac{1}{3}) \cup x \leq -2$$



الف) $y = [x] + [3-x] \rightarrow y = [x] + [-x] + 3$



ب) $y = 3x - [x]$

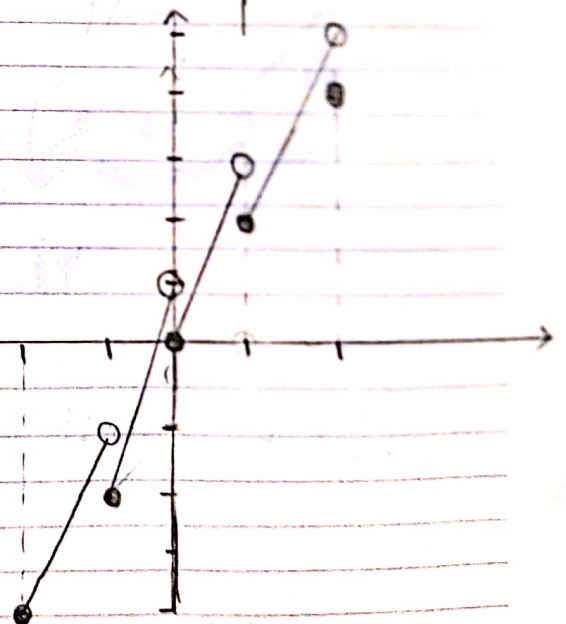
$$-2 \leq x < -1 \rightarrow [x] = -2 \rightarrow 3x + 2$$

$$-1 \leq x < 0 \rightarrow [x] = -1 \rightarrow 3x + 1$$

$$0 \leq x < 1 \rightarrow [x] = 0 \rightarrow 3x$$

$$1 \leq x < 2 \rightarrow [x] = 1 \rightarrow 3x - 1$$

$$x = 2 \rightarrow y = 4$$



$$1) y = \sin^2 x - \sin x + 1$$

$$\min = \sin x = -1 \rightarrow y = 3 \quad \sin x = 1 \rightarrow y = 1$$

$$\sin x = \frac{-b}{2a} = \frac{1}{2} \quad y = \frac{1}{4} - \frac{1}{4} + \frac{1}{4} = \frac{1}{4}$$

$$\Rightarrow R_f = [\frac{1}{4}, 3]$$

$$2) y = \sin^2 x + \sin x + 1$$

$$\min = \sin^2 x = 0 \rightarrow y = 1 \quad \max = \sin^2 x = 1 \rightarrow y = 3$$

$$\sin x = \frac{-b}{2a} = \frac{-1}{2} \rightarrow x = \frac{3\pi}{2}, \frac{\pi}{2}$$

$$\Rightarrow R_f = [1, 3]$$