

$$y = \frac{x^2 + x + 2}{x - 1} \Rightarrow yx - y = x^2 + x + 2 \Rightarrow x^2 + (1 - y)x + (y + 2) = 0 \quad R_f = (-\infty, -1] \cup [7, +\infty)$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{y - 1 \pm \sqrt{(1 - y)^2 - 4(y + 2)}}{2} \Rightarrow \Delta \geq 0 \Rightarrow (1 - y)^2 - 4(y + 2) \geq 0$$

$$y^2 - 2y + 1 - 4y - 8 \geq 0 \Rightarrow y^2 - 6y - 7 \geq 0 \quad \Delta = 64 \quad \frac{6 \pm 8}{2} \Rightarrow y = 7 \quad \frac{-1}{2} \quad \frac{7}{2}$$

الف) $2x^2 - 5x + 1$ ضابطه به شکل یک سهمی $\Delta = 25 - 4 = 21 \Rightarrow S = \frac{-\Delta}{4a} = \frac{-21}{8} \Rightarrow S = -\frac{21}{8}$

ب) $\sqrt{-x^2 + 6x - 5}$ $R = [-\frac{17}{8}, +\infty)$ $R = [0, 5]$ $S = \frac{-\Delta}{4a} = \frac{-16}{-4} = 4 \Rightarrow R = [0, 2]$

ج) $\sqrt{x^2 - 3x + 4}$ $R = [0, +\infty)$ $R = \mathbb{R}$

د) $\log(x^3 - 6x^2 + 7x + 4)$ $R = \mathbb{R}$

الف) $y = \frac{3x + 1}{2x - 6} \Rightarrow 2xy - 6y = 3x + 1 \Rightarrow (2y - 3)x = 1 + 6y \Rightarrow x = \frac{1 + 6y}{2y - 3} \Rightarrow 2y - 3 \neq 0 \Rightarrow y \neq \frac{3}{2}$

ب) $y = \sqrt{\frac{x + 1}{x - 2}} \Rightarrow y^2 = \frac{x + 1}{x - 2} \Rightarrow y^2(x - 2) = x + 1 \Rightarrow (y^2 - 1)x = 1 + 2y^2 \Rightarrow x = \frac{1 + 2y^2}{y^2 - 1} \Rightarrow R = \mathbb{R} - \{\frac{1}{2}\}$

ج) $\sqrt{x} \Rightarrow R = [0, +\infty) - \{2\}$

الف) $y = \frac{3x^2 + 4}{x^2 - 1} \Rightarrow yx^2 - y = 3x^2 + 4 \Rightarrow (y - 3)x^2 = 4 + y \Rightarrow x = \sqrt{\frac{4 + y}{y - 3}} \Rightarrow \frac{-4}{y - 3} \quad \frac{3}{y - 3}$

ب) $x^2 = 0 \Rightarrow \frac{3x^2 + 4}{x^2 - 1} = \frac{4}{-1} = -4$ $x^2 \neq -\infty \Rightarrow \frac{3x^2 + 4}{x^2 - 1} = 3 \Rightarrow \frac{3x^2}{x^2 - 1} = 3$ $R = (-\infty, -4] \cup [3, +\infty)$

الف) $y = \frac{2\sin x - 1}{\sin x + 3} \Rightarrow y \sin x + 3y = 2\sin x - 1 \Rightarrow (y - 2)\sin x = -3y - 1 \Rightarrow \sin x = \frac{-3y - 1}{y - 2}$

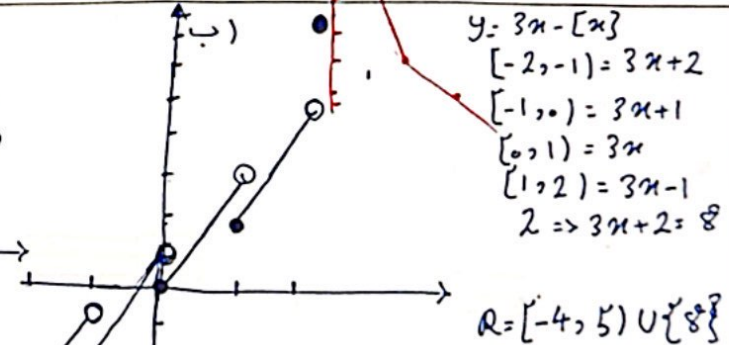
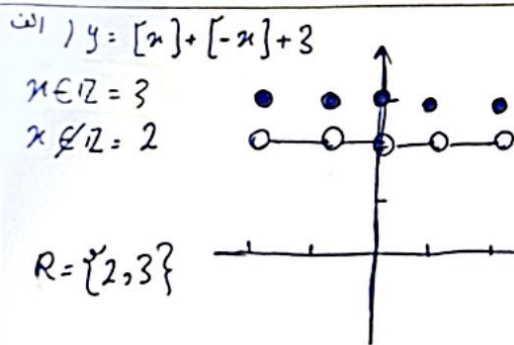
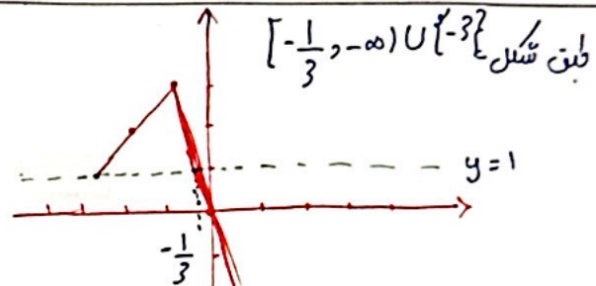
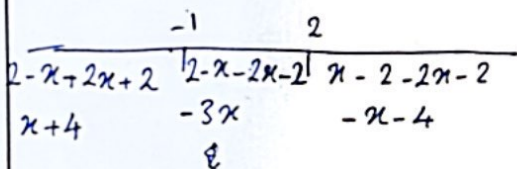
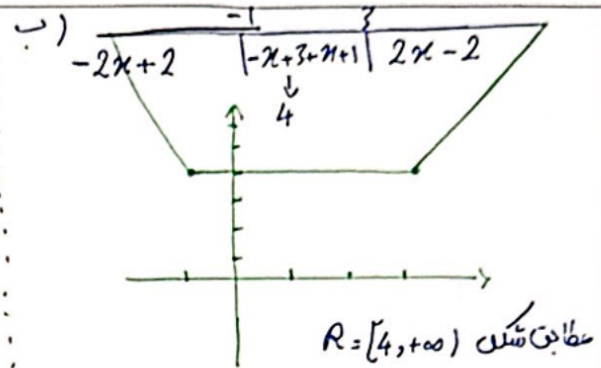
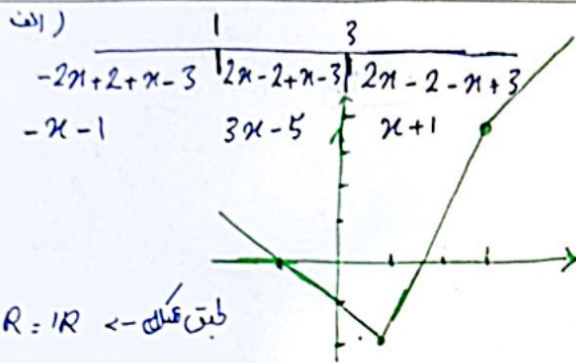
$-1 \leq \frac{-3y - 1}{y - 2} \leq 1 \Rightarrow 2 - y \leq -3y - 1 \Rightarrow 3 \leq -2y \Rightarrow \frac{3}{2} \geq y$ $-3y - 1 \leq y - 2 \Rightarrow 1 \leq 4y \Rightarrow \frac{1}{4} \leq y$

ب) $\sin x = 1 \Rightarrow \frac{2 - 1}{1 + 3} = \frac{1}{4}$ $\sin x = -1 \Rightarrow \frac{-3}{2}$ $R = [-\frac{3}{2}, \frac{1}{4}]$

جواب: $x^2 - x + 1 \neq 0 \Rightarrow \Delta = b^2 - 4ac \rightarrow 1 - 4 = -3 \rightarrow \Delta < 0 \rightarrow$ لا يوجد حلا
 $D_f = \mathbb{R}$

ب) $y: \frac{x^2 - x + \frac{1}{4}}{x^2 - x + 1} \Rightarrow \frac{(x - \frac{1}{2})^2}{(x - \frac{1}{2})^2 + \frac{3}{4}} = y \Rightarrow (x - \frac{1}{2})^2 = y(x - \frac{1}{2})^2 + \frac{3y}{4} \rightarrow (1 - y)(x - \frac{1}{2})^2 = \frac{3y}{4}$

$(x - \frac{1}{2})^2 = \frac{3y}{4(1 - y)} \Rightarrow x - \frac{1}{2} = \sqrt{\frac{3y}{4(1 - y)}} \Rightarrow x = \sqrt{\frac{3y}{4(1 - y)}} + \frac{1}{2} \Rightarrow y = \frac{0}{-1} + \frac{1}{-1} = -1$
 $R = [0, 1]$



الف) $y = \sin^2 x - \sin x + 1$
 $\sin x = 1 \Rightarrow y = 1$
 $\sin x = -1 \Rightarrow y = 3$
 $\sin x = \frac{1}{2} \Rightarrow y = \frac{3}{4}$

$R = [1, 3]$

ب) $y = \sin^4 x + \sin^2 x + 1$
 $\sin^2 x = 0 \Rightarrow y = 1$
 $\sin^2 x = 1 \Rightarrow y = 3$
 $\sin^2 x = -\frac{1}{2} \Rightarrow \text{لا يوجد}$

$R = [1, 3]$