شکوه حیدری

بسم الله الرحمن الرحیم

کلاس: سیزدهم‌ها (۱۹-۲۱) — تکلیف (۴) چهار

سوال ۱

الف) $y=(x-2[\frac{x}{2}])^2=(2(\frac{x}{2}-[\frac{x}{2}]))^2 \Rightarrow 0\le \omega-[\omega]<1$

$(2(\omega-[\omega]))^2=(0\le 2\omega-2[\omega]<2)^2 \Rightarrow R_f=[0,4)$

ب) $\sin x+\log y-4=0 \Rightarrow \log y=4-\sin x \Rightarrow y=10^{4+\sin x}$

$-1\le \sin x\le 1 \Rightarrow 3\le 4+\sin x\le 5 \quad 10^3\le 10^{4+\sin x}\le 10^5 \Rightarrow R=[10^3,10^5]$

سوال ۲

$f(x)=\sqrt{\frac{x^2-4}{x^2-9}} \Rightarrow \min x=0 \Rightarrow \frac{x^2-4}{x^2-9}=\frac{4}{9}$

$\max x=+\infty \Rightarrow \frac{+\infty^2}{+\infty^2}=1$ برد صورت $R=(-\infty,\frac{4}{9}]\cup(1,+\infty)$

$\sqrt{(-\infty,\frac{4}{9}]\cup(1,+\infty)} \Rightarrow [0,\frac{2}{3}]\cup(1,+\infty) \Rightarrow 0+\frac{2}{3}+1=\frac{5}{3}$

سوال ۳

الف) $f(x)=x+\frac{1}{x}+2\sqrt{x+\frac{1}{x}} \Rightarrow (\sqrt{x+\frac{1}{x}}+1)^2-1$

$\Rightarrow R_f=(2+2\sqrt{2},+\infty)$

ب) $(3-\sin x)(1+\sin x)=3-\sin x+3\sin x-\sin^2 x=$

$-\sin^2 x+2\sin x+3 \xrightarrow{\sin x=1} y=4$

$\xrightarrow{-1} y=0$

$-\frac{b}{2a}=\frac{-2}{-2}=1=0$

$R_f=[0,4]$

سوال ۴

$D_f = \{1, 2, 3, \dots, 9\} \Rightarrow f(x) = -\frac{1}{3}x + 10$

$R_f = \left\{ -\frac{1}{3} + 10, -\frac{2}{3} + 10, -\frac{3}{3} + 10, \dots, -\frac{9}{3} + 10 \right\} \Rightarrow$

$-\frac{(1+2+\dots+9)}{3} + 9(10) \Rightarrow -\frac{\left(\frac{9 \times 10}{2}\right)}{3} + 90 = -\frac{45}{3} + 90 = \boxed{75}$

سوال ۵

$\sqrt{ax^2+bx+c} \to D_f = [2, +\infty)$

ریشه ?

$\begin{array}{c|c} & 2 \\ \hline - & + \end{array} \Rightarrow a = 0 \quad \mathrm{I}$

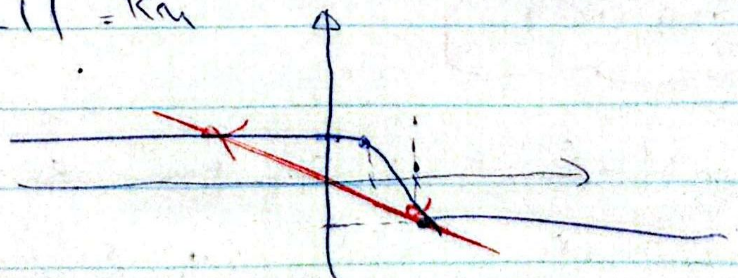
$2b + c = 0 \quad \mathrm{II}$

$f(3) = 1 \to \left.\begin{array}{l} 3b + c = 1 \\ 2b + c = 0 \end{array}\right\} \to b = 1, \; c = -2$

$\sqrt{bx^2 + ax - 2c} = \sqrt{x^2 + 4} \Rightarrow \sqrt{[4, +\infty)} \Rightarrow R_f = [2, +\infty)$

سوال ۶

$|x-2| - |x-1| = kx$



$kx \xrightarrow{(2,-1)} 2k = -1 \Rightarrow \boxed{k = -\frac{1}{2}}$

سوال ۷

الف) $x[x]$

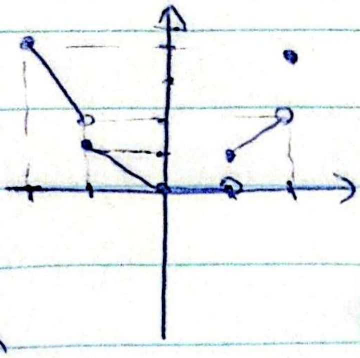
$-2\le x<-1 \xrightarrow{[x]=-2} -2x$

$-1\le x<0 \xrightarrow{[x]=-1} -x$

$0\le x<1 \xrightarrow{[x]=0} =0$

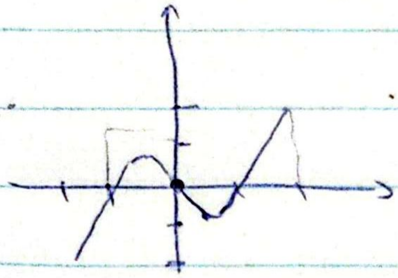
$1\le x<2 \xrightarrow{[x]=1} x$

$2\le x<3 \xrightarrow{[x]=2} 2x$



ب) $x|x|-x$

$x(|x|-1)=$ $-1 \quad 0 \quad 1$: $-x-x^2 \quad -x^2-x \quad x^2-x \quad x^2-x$



سوال ۸

$$y=\frac{1}{x}+\frac{1}{x+1}+\frac{1}{x^2+x} \Rightarrow \frac{2x+2}{x^2+x}=\frac{2\cancel{(x+1)}}{x\cancel{(x+1)}}$$

$\Rightarrow \frac{2}{x} \Rightarrow R=\mathbb{R}-\{0\}$ ①

$D_f=\mathbb{R}-\{0,-1\}$

$x\neq-1 \Rightarrow y\neq-2$ ②

①,② $\Rightarrow R_f=\mathbb{R}-\{0,-2\} \Rightarrow 0+(-2)=\boxed{-2}$

سوال ۹

$$f(x)=\frac{x-4\sqrt{x}+3}{x-2\sqrt{x}+1} \Rightarrow \frac{\cancel{(\sqrt{x}-1)}(\sqrt{x}-3)}{\cancel{(\sqrt{x}-1)}(\sqrt{x}-1)}=\frac{\sqrt{x}-3}{\sqrt{x}-1}$$

$\min x=0 \longrightarrow y=3$

$\max x=+\infty \longrightarrow y=1$

مخرج ریشه دارد $R_f=(-\infty,1)\cup(3,+\infty)$

سوال ۱۰

$$f(x)=\frac{x^3+2x^2-x-2}{x^2-1}=\frac{x(x^2-1)+2(x^2-1)}{x^2-1}$$

$$\frac{\cancel{(x^2-1)}(x+2)}{\cancel{(x^2-1)}} \Rightarrow D_f=\mathbb{R}-\{1,-1\} \Rightarrow R_f=\mathbb{R}-\{1,3\}$$

$1 \to 3 \qquad -1 \to 1$

$1+3=\boxed{4}$