

الف) $y = \left(x - \sqrt{\frac{x}{2}}\right)^2 \rightarrow \left(\sqrt{\frac{x}{2}} - \sqrt{\frac{x}{2}}\right)^2 \xrightarrow{\frac{x}{2} = t} (\sqrt{2t} - \sqrt{2t})^2$
 $R = [0, 2]^2$

$\Rightarrow R_y = [0, 2]$ ✓

ب) $\text{Cev } y = 1 + \sin x \Rightarrow y = 1 + \sin x$
 $R_y = [1, 2]$ ✓

$f(x) = \sqrt{\frac{x^2 - 4}{x^2 - 1}} \rightarrow R = [0, \frac{2}{3}] \cup (1, +\infty) = [0, \frac{2}{3}] \cup (1, +\infty)$

$\Rightarrow a = 0, b = \frac{2}{3}, c = 1 \rightarrow a + b + c = \frac{5}{3}$ ✓

الف) $f(x) = 2x + \frac{1}{x} + \sqrt{x+1} \Rightarrow R_f = [2, +\infty)$
 $[2, +\infty) \quad [2, +\infty)$

برای $\min$ به ازای $x=1$ ،
 ایستایی داریم.

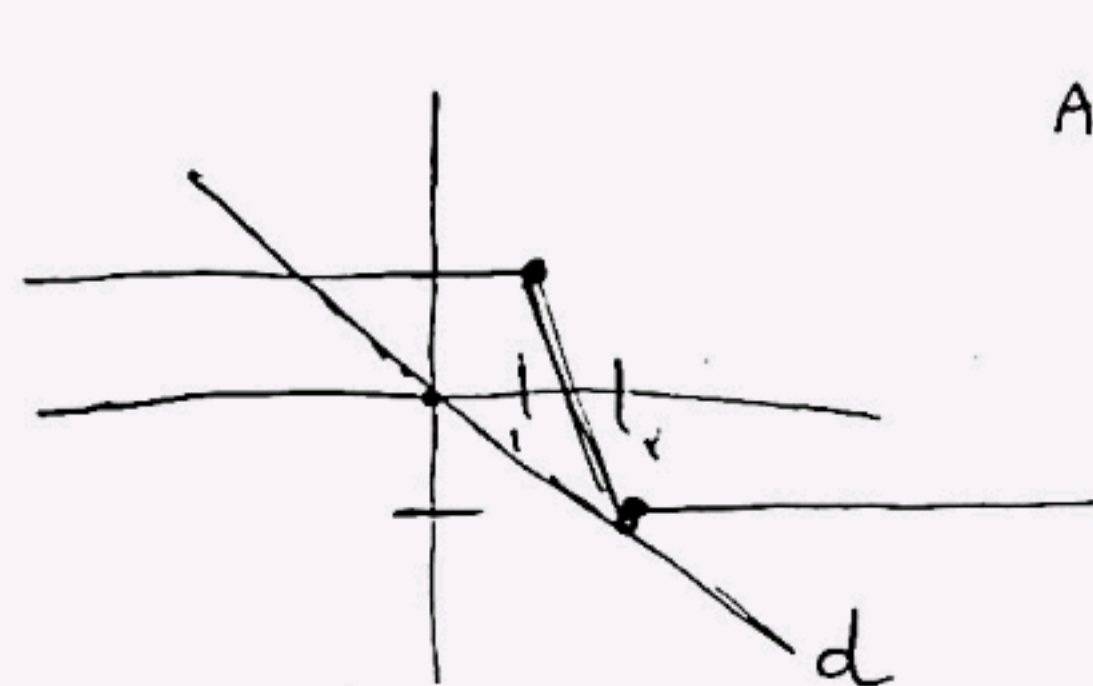
ب) $f(x) = (3 - \sin x)(1 + \sin x) = y = -\sin^2 x + 3\sin x + 3$
 $\sin x = 1 \rightarrow 4$
 $\sin x = -1 \rightarrow 0$
 $\sin x = \frac{3}{2} \rightarrow \text{not possible}$
 $\Rightarrow R = [0, 4]$ ✓

$f(x) = -\frac{1}{x}x + 10, D_f = \{x \mid x \in \mathbb{N}, 1 \leq x \leq 9\}$

$f(x) = \left\{ (1, \frac{19}{1}), (2, \frac{18}{2}), (3, 9), (4, \frac{17}{4}), (5, \frac{16}{5}), (6, 8), (7, \frac{15}{7}), (8, \frac{14}{8}), (9, 7) \right\}$
 $\rightarrow \text{مجموع اعداد} = \frac{125}{3} = 41\frac{2}{3}$ ✓

$f(x) : \Rightarrow a = 0$
 $x = 2 \rightarrow 2b + c = 0$
 $x = 3 \rightarrow 3b + c = 1 \Rightarrow b = 1, c = -2$
 $g(x) = \sqrt{bx^2 + ax + c} = \sqrt{x^2 + 1} \rightarrow R_g = [1, +\infty)$ ✓

$$|x-r| - |x-1| = kx$$

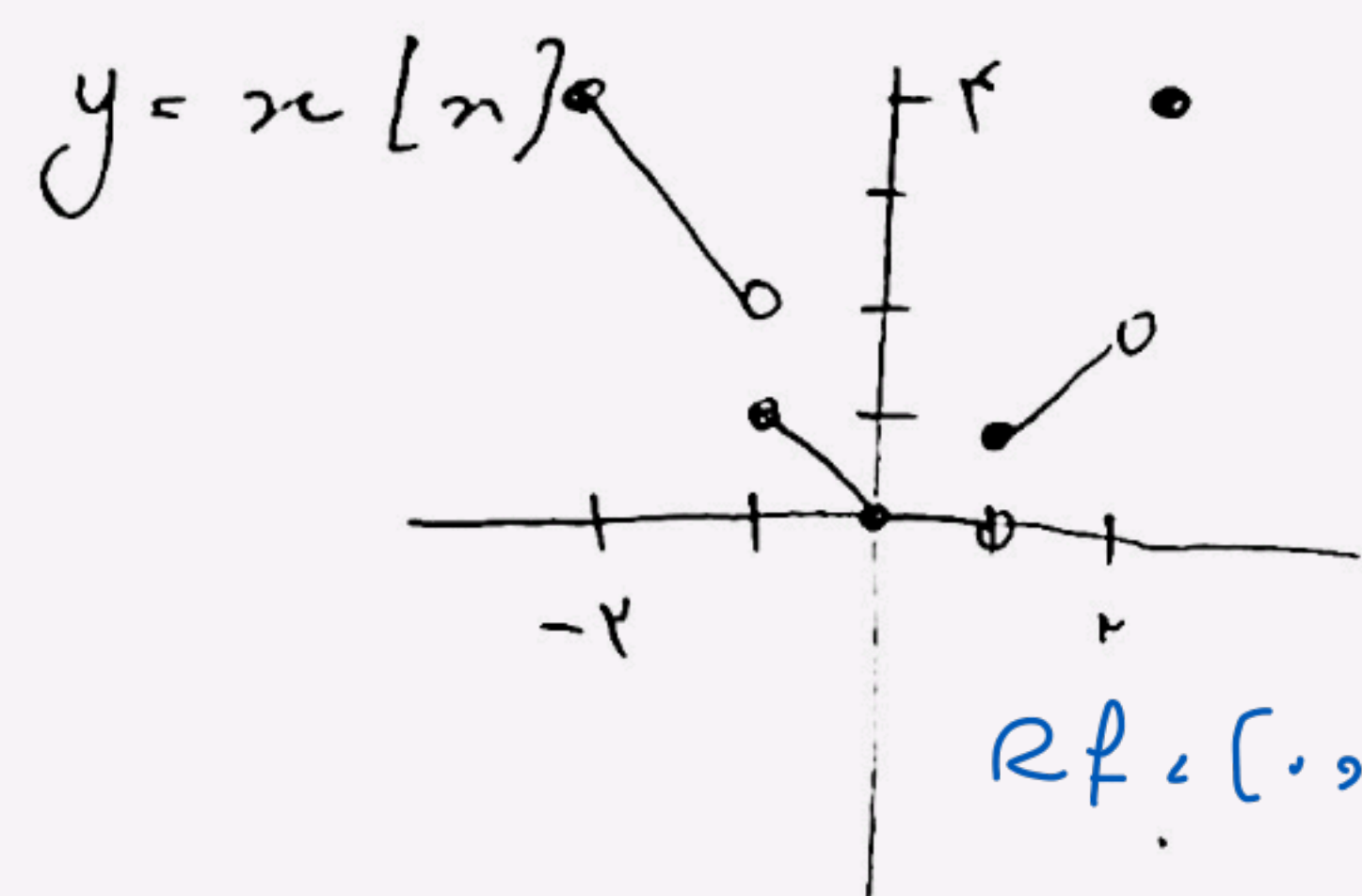


$$A|0$$

$$a = \frac{1}{-r}$$

$$B| \begin{matrix} r \\ -1 \end{matrix} d \Rightarrow \boxed{k = -\frac{1}{r}}$$

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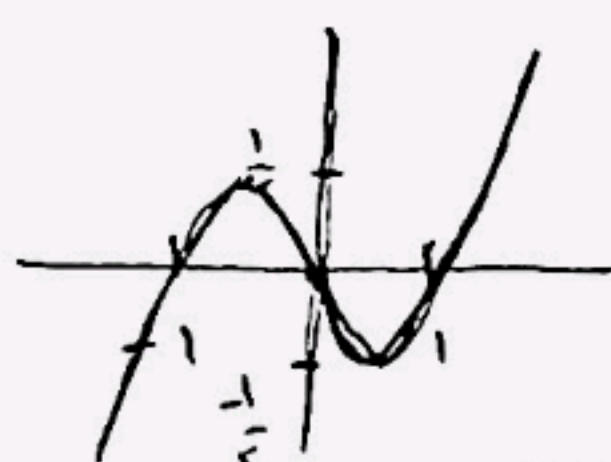


$$Df = [0, \infty) \cup (-\infty, 0]$$

$$y = x|x| - x$$

$$x > 0 \rightarrow x^2 - x$$

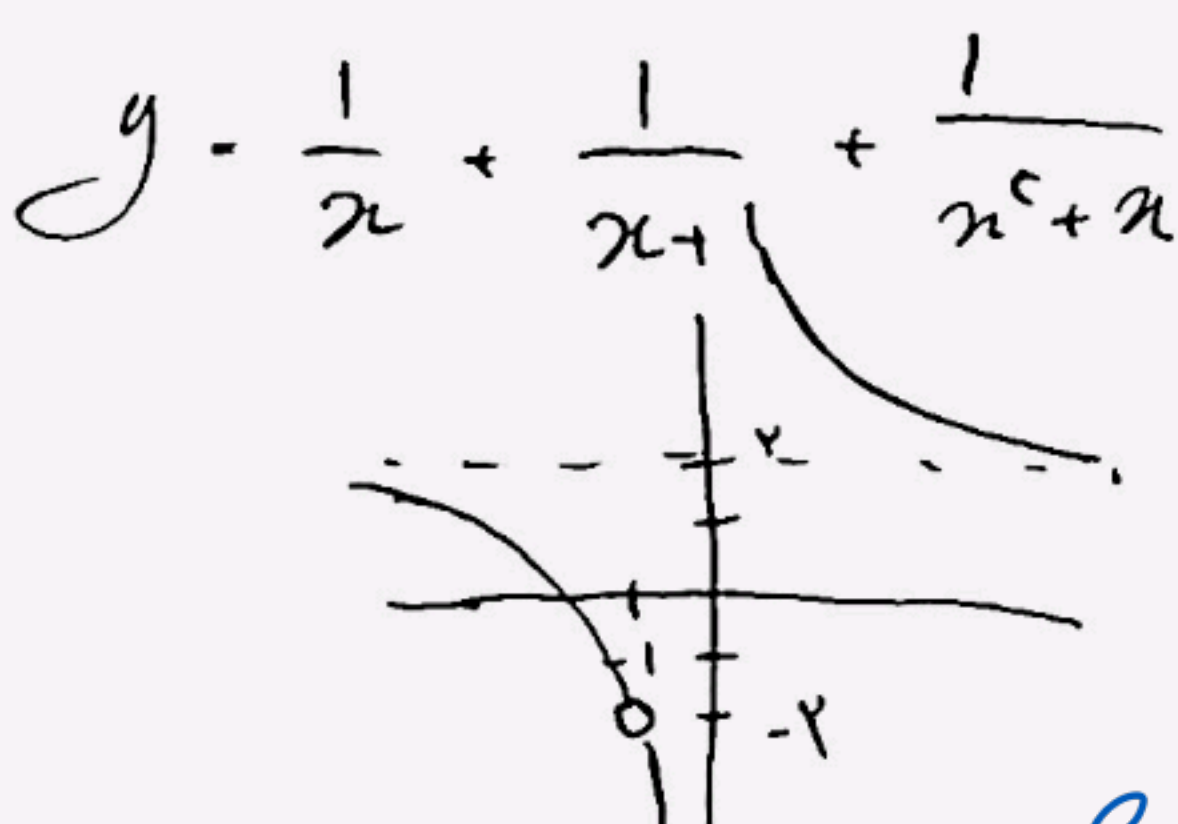
$$x < 0 \rightarrow x^2 - x$$



$$Dy = [-1, 0] \cup [0, 1]$$

1, 2

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$$Df = \mathbb{R} - \{0, -1\}$$

$$\Rightarrow R = \mathbb{R} - \left\{ -1, \frac{1}{2} \right\}$$

$$Df = \mathbb{R} - \left\{ 0, \frac{1}{2} \right\}$$

$$\boxed{a+b=0}$$

$$a < 0 \rightarrow a+b < -r$$

1, 2

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$$f(x) = \frac{x - \varepsilon \sqrt{x^2 + r}}{x - r \sqrt{x^2 + 1}} \approx \frac{(\sqrt{x} - r)(\sqrt{x} - 1)}{(\sqrt{x^2} - 1)(\sqrt{x^2} - 1)} \xrightarrow{x \neq 1} \frac{\sqrt{x^2} - r}{\sqrt{x^2} - 1} = f(x)$$

$$R = (-\infty, 1) \cup [r, +\infty)$$

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$$f(x) = \frac{x^r + r x^r - x - r}{x^r - 1} = \frac{(x-r)(x+1)(x+r)}{(x-1)(x+1)} = x+r$$

$$x \neq 1 \rightarrow y \neq r$$

$$x = -1 \rightarrow y \neq 1$$

$$\Rightarrow R = \mathbb{R} - \{r, 1\}$$

$$\boxed{\begin{matrix} \frac{1}{x} = 4r \\ \frac{1}{x} = r \end{matrix}}$$

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