دوازدهم دختر جمعه — تکلیف شماره ۴ — هنرنیا ریاضی

$y=\left(x-2\left[\frac{x}{2}\right]\right)^2 \to \left(2\left(\frac{x}{2}-\left[\frac{x}{2}\right]\right)\right)^2 \to R=[0,4)$ ۱

$0\le \frac{x}{2}-\left[\frac{x}{2}\right]<1 \quad 0\le 2\left(\frac{x}{2}-\left[\frac{x}{2}\right]\right)<2$

$-\sin x+\log y-4=0 \quad \log y=\sin x+4$

$y=10^{\sin x+4}$ $\xrightarrow{\sin x=-1} y=10^3$ $\quad R=[10^3,10^5]$

$\xrightarrow{\sin x=1} y=10^5$

$f(x)=\sqrt{\frac{x^2-4}{x^2-9}} \quad y^2=\frac{x^2-4}{x^2-9}=\frac{x^2-9+5}{x^2-9}=1+\frac{5}{x^2-9}$ ۲

$x^2=\frac{5}{y^2-1}+9 \quad x=\sqrt{\frac{9y^2-4}{y^2-1}}=\sqrt{\frac{(3y-2)(3y+2)}{(y-1)(y+1)}}$ $\quad -1 \quad \frac{-2}{3} \quad \frac{2}{3} \quad 1$: $+ \; - \; + \; - \; +$

$\left[\frac{-2}{3},\frac{2}{3}\right]\cup(1,+\infty) \quad a+b+c=1$

$f(x)=x+\frac{1}{x}+2\sqrt{x+\frac{1}{x}}+1-1=\left(\sqrt{x+\frac{1}{x}}+1\right)^2-1$ $\quad R$ ۳

$y=\left(\sqrt{x+\frac{1}{x}}+2\right)\left(\sqrt{x+\frac{1}{x}}\right) \quad y=t^2+2t \quad y=\sqrt{b+1}-1$

$t=\sqrt{x+\frac{1}{x}} \quad t=y^2+2y \quad b+1=(y+1)^2$

$y=(3-\sin x)(1+\sin x)=-\sin^2 x+2\sin x+3$

$\xrightarrow{\sin x=-1} -1-2+3=0$

$\xrightarrow{\sin x=1} -1+2+3=4 \quad R=[0,4]$

$\frac{-b}{2a} \to \frac{-2}{-2}=1$ [illegible]

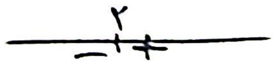
$f(x) = \frac{-1}{3}x + 10$ $\qquad D_f = \{1,2,3,4,5,6,7,8,9\}$ ۴

$R_f = \{\frac{29}{3}, \frac{28}{3}, 9, \frac{26}{3}, \dots, 7\}$

$f(x) = \sqrt{ax^2+bx+c}$ $\qquad f(3)=1$ $\qquad g(x) = \sqrt{bx^2+ax-2c}$ ۵

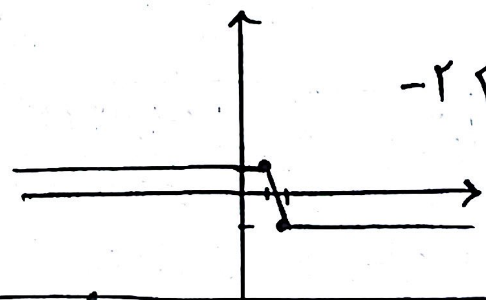
$a=0$ $\qquad bx+c \xrightarrow{x=3} 3b+c=1$ $\qquad b=1 \quad c=-2$

$\qquad\qquad bx+c \xrightarrow{x=2} 2b+c=0$



$y = \sqrt{x^2+4}$ $\qquad R = [2, +\infty)$ $\qquad g(x) = \sqrt{x^2+4}$

$|x-2| - |x-1| = kx$ $\qquad -2 < k < 0$ ۶



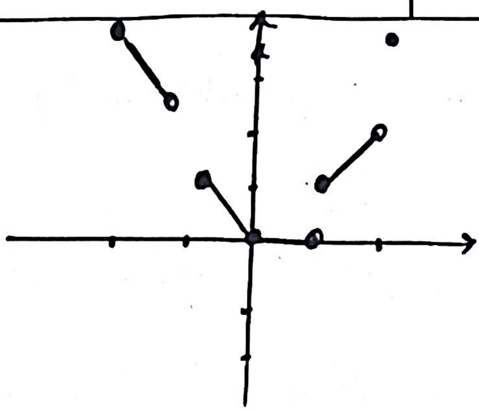
$y = x[x]$ ۷

$-2 \leq x < -1 \to -2x$

$-1 \leq x < 0 \to -x$

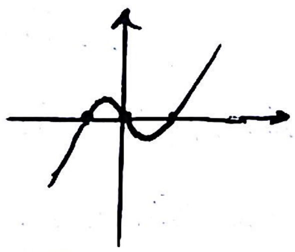
$0 \leq x < 1 \to 0$

$1 \leq x < 2 \to x$



$y = x|x| - x$

$$\begin{cases} x^2 - x & x \geq 0 \\ -x^2 - x & x < 0 \end{cases}$$



$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x}$ $\qquad \mathbb{R} - \{a, b\}$ $\qquad a+b=-1$ ۸

$\frac{x+1+x+1}{x(x+1)} = \frac{2(x+1)}{x(x+1)} = \frac{2}{x}$ $\qquad R_f = \mathbb{R} - \{0, -1\}$

$$f(x)=\frac{x-۴\sqrt{x}+۳}{x-۲\sqrt{x}+۱}=\frac{(\sqrt{x}-۱)(\sqrt{x}-۳)}{(\sqrt{x}-۱)(\sqrt{x}-۱)}=\frac{(\sqrt{x}-۳)}{(\sqrt{x}-۱)}$$ ۹

$$y=\frac{\sqrt{x}-۳}{\sqrt{x}-۱}\qquad \sqrt{x}=\frac{-y+۳}{-y+۱}\longrightarrow \sqrt{y}=\frac{-x+۳}{-x+۱}=\frac{۳-x}{۱-x}$$

$$\frac{۳-x}{۱-x}\geq ۰\qquad \begin{array}{ccccc} & ۱ & & ۳ & \\ - & \text{ن} & + & ۰ & - \end{array}\qquad D_{f^{-1}}=R_f=(۱,۳]$$

$$f(x)=\frac{x^۳+۲x^۲-x-۲}{(x-۱)(x+۱)}=\frac{x^۲(x-۱)+۲(x^۲-۱)}{(x-۱)(x+۱)}=\frac{(x-۱)(x^۲+۲x+۲)}{(x-۱)(x+۱)}$$ ۱۰

$$f(x)=\frac{x^۲+۲x+۲}{x+۱}=\frac{(x+۱)^۲+۱}{(x+۱)}=x+۱+\frac{۱}{x+۱}$$

$$R_f=\mathbb{R}-\left\{-۱,\frac{۵}{۲}\right\}\qquad -۱+\frac{۵}{۲}=\frac{۳}{۲}$$