

خاکه رستمان - دوزده

الف) $y = (x - 2[\frac{x}{2}])^2 \rightarrow y = (2(\frac{x}{2} - [\frac{x}{2}]))^2$

وال ۱

$0 \leq x - 2[\frac{x}{2}] < 1$

$0 \leq \frac{x}{2} - [\frac{x}{2}] < \frac{1}{2}$

$K_f = [0, 1]$

ب) $\log y = \sin x + 1 \rightarrow y = 10^{\sin x + 1}$ $-1 \leq \sin x \leq 1$

$y = 10^{1+1} = 10^2$
 $y = 10^{-1+1} = 10^0$

$K_f = [10^0, 10^2]$

$y = \sqrt{\frac{x^2-4}{x^2-9}}$

$y = \frac{x^2-4}{x^2-9} \rightarrow yx^2 - 4y = x^2 - 9 \rightarrow x^2 = \frac{4y-9}{y-1}$

وال ۲

$yx^2 - x^2 = 4y - 9 \rightarrow x^2(y-1) = 4y-9 \rightarrow x^2 = \frac{4y-9}{y-1} \rightarrow x = \pm \sqrt{\frac{4y-9}{y-1}}$

$\frac{4y-9}{y-1} \geq 0$ $\frac{4}{y-1}$ $\frac{9}{y-1}$ $\frac{1}{y-1}$ $\rightarrow [0, \frac{9}{4}] \cup (1, +\infty)$

$a = 0$

$b = \frac{9}{4}$

$a+b+c = \frac{9}{4} + \frac{9}{4} = \frac{9}{2}$

$c = 1$

الف) $y = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}}$

رابطه دافه
زیر ابدال
معر

$x + \frac{1}{x} \geq 0$

$\frac{x^2+1}{x} \geq 0 \rightarrow x > 0$
مقدار همواره مثبت

$K_f = [1, 2\sqrt{2} + \infty)$

$x + \frac{1}{x} = -2$

$x + \frac{1}{x} = 2$
حل مقبول

$(-\infty, -1) \cup (1, +\infty)$
مقدار خارج از دامنه

ب) $y = (1 - \sin x)(1 + \sin x) \rightarrow y = 1 - \sin^2 x$

$y = -\sin^2 x + \sin x + 1$ $\xrightarrow{\sin x = 1} -1 + 1 + 1 = 1$

$\xrightarrow{\sin x = -1} -1 - 1 + 1 = -1$
 $\xrightarrow{\sin x = \frac{1}{2}} -\frac{1}{4} + \frac{1}{2} + 1 = \frac{5}{4}$

$K_f = [0, 1]$

$$f(x) = -\frac{1}{p}x + b$$

$$0 = 1, 2, 3, 4, 5, 6, 7, 8, 9$$

سوال ۴

$$x=1 \rightarrow -\frac{1}{p} + 1 = \frac{p-1}{p}$$

$$x=2 \rightarrow -\frac{1}{p} \times 2 + 1 = -\frac{2}{p} + 1 = \frac{p-2}{p}$$

1

$$x=9 \rightarrow -\frac{1}{p} \times 9 + 1 = -\frac{9}{p} + 1 = \frac{p-9}{p}$$

$$مجموع = \frac{(1 + 2 + \dots + 9) \times \frac{p-1}{p}}{9}$$

$$\left(\frac{1}{p} + \frac{2}{p} + \dots + \frac{9}{p} \right) \times 9 =$$

$$\frac{1+2+\dots+9}{p} \times 9 = \frac{45}{p} \times 9 = 405$$

سوال ۵

$$f(x) = \sqrt{ax^2 + bx + c}$$

$$f(0) = \sqrt{c} \rightarrow \text{نقطه یابی}$$

$$a=0 \rightarrow \text{نقطه یابی}$$

$$2b + c = 0$$

$$2b + c = 1$$

$$\rightarrow b=1$$

$$2 + c = 1 \rightarrow c = -1$$

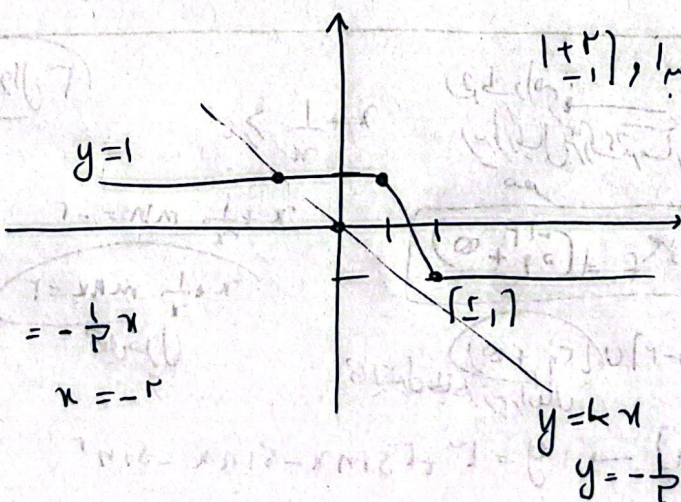
$$g(x) = \sqrt{x^2 + 0(x) - 2x - 1} \rightarrow y = \sqrt{x^2 - 2x - 1}$$

$$\min x^2 - 2x - 1 \rightarrow \min y = \sqrt{4 - 2} = \sqrt{2}$$

$$(K_f = \sqrt{29 + 0})$$

$$|x-2| - |x-1| = kx$$

سوال ۶



$$1 = -\frac{1}{p}x$$

$$x = -p$$

$$y = kx$$

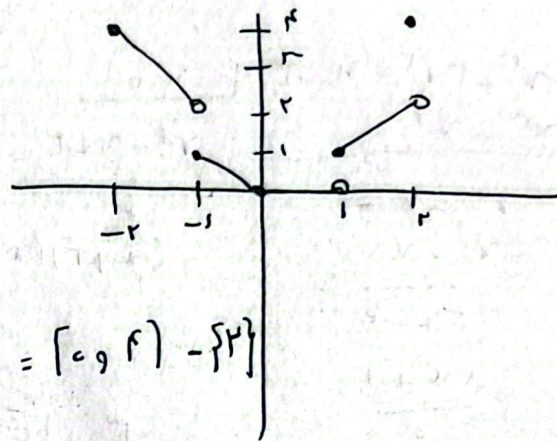
$$y = -\frac{1}{p}x$$

$$y = kx \xrightarrow{x=-1} y=-1 \rightarrow k = -1$$

$$k = -\frac{1}{p}$$

الف) $y = x|x|$

$-2 \leq x < -1 \rightarrow |x| = -x \rightarrow -2x$
 $-1 \leq x < 0 \rightarrow |x| = -1 \rightarrow -x$
 $0 \leq x < 1 \rightarrow |x| = 0 \rightarrow 0$
 $1 \leq x < 2 \rightarrow |x| = 1 \rightarrow x$
 $x = 2 \rightarrow |x| = 2 \rightarrow 2x$

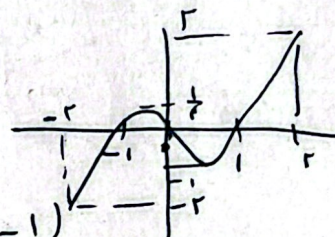


$K_f = [-2, 2] - \{0\}$

ب) $y = x|x| - x$

$0 \leq x \leq 1 \rightarrow y = x^2 - x \rightarrow y = x(x-1)$

$-1 \leq x < 0 \rightarrow y = -x^2 - x \rightarrow y = -x(x+1)$



$-\frac{b}{2a} = -\frac{1}{-2} = \frac{1}{2} \rightarrow -\frac{1}{4} + \frac{1}{4} = 0$

$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} \rightarrow \frac{x+1+x+1}{x^2+x}$

$\frac{x^2+2x+2}{x^2+x} \rightarrow y = \frac{x^2+2x+2}{x(x+1)} \rightarrow \frac{x(x+1)+1}{x(x+1)} = \frac{1}{x} \rightarrow x \neq -1$

$K = \{-1, 0\} \rightarrow a+b = -1+0 = -1$

$f(x) = \frac{x - \sqrt{x+1}}{x - \sqrt{x} + 1} \rightarrow \frac{(\sqrt{x}-1)(\sqrt{x}-1)}{(\sqrt{x}-1)\sqrt{x}} = \frac{(\sqrt{x}-1)}{(\sqrt{x}-1)}$

$y = \frac{\sqrt{x}-1}{\sqrt{x}-1} \rightarrow y\sqrt{x} - y = \sqrt{x} - 1 \rightarrow y\sqrt{x} - \sqrt{x} = y - 1$

$\sqrt{x} = \frac{y-1}{y-1} \rightarrow \sqrt{x} = 1 \rightarrow x = 1$

وال ١٠

$$y = \frac{x^5 + 2x^2 - x - 5}{x^2 - 1} \rightarrow \text{بجای } 0 \text{ بگذاریم } x+1$$

$$\begin{array}{r} x^5 + 2x^2 - x - 5 \\ \underline{-x^5 - x^3} \\ -x^3 + 2x^2 - x - 5 \end{array}$$

$$\begin{array}{r} -x^3 + 2x^2 - x - 5 \\ \underline{-x^3 - x} \\ 2x^2 - 2x - 5 \end{array}$$

$$\begin{array}{r} 2x^2 - 2x - 5 \\ \underline{-2x^2 - 2} \\ -2x - 3 \end{array}$$

$$x^2 + x - 2$$

$$\downarrow$$

$$(x+2)(x-1)$$

$$x+1 = \textcircled{2}$$

$$y = \frac{(x+1)(x+2)(x-1)}{(x-1)(x+1)}$$

$$x \neq 1, -1$$

$$y = x+2$$

$$-1+2 = \textcircled{+1}$$

$$1+2 = \textcircled{+3}$$

