تمرین تئوری:

1) $y=(x-2[\frac{x}{2}])^2=(2(\frac{x}{2}-[\frac{x}{2}]))^2 \Rightarrow 0\le \frac{x}{2}-[\frac{x}{2}]<1 \xrightarrow{\times 2} 0\le \square <2 \xrightarrow{\text{توان ۲}} 0\le \square^2<4 \Rightarrow R=[0,4)$

ب) $\log y=\sin x+4 \to \log y-4=\sin x \Rightarrow -1\le \log y-4\le 1 \Rightarrow 3\le \log y\le 5 \Rightarrow y\in[10^3,10^5]$

2) $f(x)=\sqrt{\frac{x^2-4}{x^2-9}} \Rightarrow \max\to\infty \Rightarrow \frac{\infty}{\infty}=1$

$\min \Rightarrow 0$

$R=\mathbb{R}-\{\frac{4}{9}\} \Rightarrow$ مخرج ریشه دارد $\Rightarrow$

ندارد ندارد

$\sqrt{\frac{4}{9}}$

$R_f=[0,\frac{2}{3}]\cup(1,+\infty) \Rightarrow a+b+c=\frac{2}{3}+1=\frac{5}{3}$

از (۱) تا بی‌نهایت بوده اولیه

بازه باز است

3) الف) $a+\frac{1}{a}\ge 2 \le -2 \to \sqrt{a+\frac{1}{a}}\ge\sqrt{2} \to x+\frac{1}{x}+2\sqrt{x+\frac{1}{x}} \Rightarrow \min\to x=1 \to 1+1+2\sqrt{2}$

$\Rightarrow 2+2\sqrt{2} \to R=[2+2\sqrt{2},+\infty)$

ب) $3+3\sin x-\sin x-\sin^2 x \Rightarrow -\sin^2 x+2\sin x+3$

$-1 \to -1-2+3=0$

$1 \to -1+2+3=4$

$-\frac{b}{2a} \Rightarrow \frac{-2}{-2}=-1$

$R\Rightarrow[0,4]$

4) $-\frac{1}{3}x+10,\ -\frac{2}{3}x+10,\ -\frac{3}{3}x+10,\dots,-\frac{9}{3}x+10 \Rightarrow 10\times 9+(-\frac{1}{3}+-\frac{2}{3}+-\frac{3}{3},\dots,-\frac{9}{3})=90-15=75$

$a=-\frac{1}{3}$, $n=9$, $a_n=-\frac{9}{3} \Rightarrow \frac{9}{2}(-\frac{10}{3})=-15$

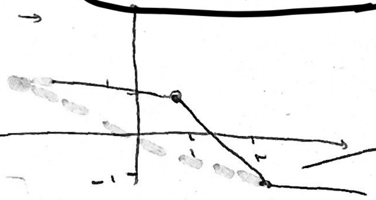
5) الف $\Rightarrow$ معادله یک خط است

$a=0 \Rightarrow \sqrt{bx+c} \Rightarrow 2b+c=0$, $3b+c=1 \Rightarrow b=1$, $c=-2$

ب) $\sqrt{x^2+4} \Rightarrow \sqrt{x^2+4}=y \to R=[2,+\infty)$

$\min \to x=0 \to \sqrt{4}=2$

6) $|x-2|-|x-1|=k \to$



$kx \leftarrow$ مبدأ گذر است

$(2,-1),(0,0) \Rightarrow$ شیب $=\frac{0-(-1)}{0-2}=-\frac{1}{2}=k$

7)

$[-2,-1) \to -2x$

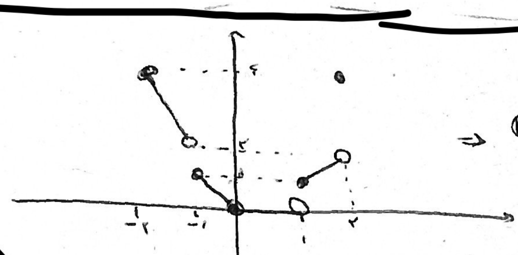
$[-1,0) \to -x$

$[0,1) \to 0$

$[1,2) \to x$

$[2]=4$

$\Rightarrow$



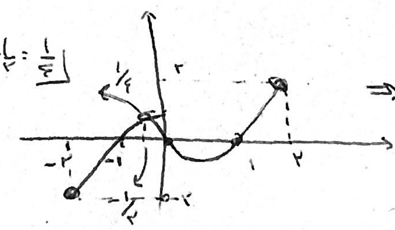
$\Rightarrow R\in[0,4]-\{2\}$

ب) $x|x|-x$

$x\in[-2,0] \Rightarrow -x^2-x$

$x\in[0,2] \Rightarrow x^2-x$

$-(\frac{1}{4})+\frac{1}{2}=\frac{1}{4}$



$\Rightarrow R\in[-2,2]$

۸) $\dfrac{x+1+x}{x^2+x} + \dfrac{1}{x^2+x} \Rightarrow \dfrac{2x+2}{x^2+x} \Rightarrow \dfrac{2(x+1)}{x(x+1)} = y \Rightarrow \overset{x+1\neq 0}{\Rightarrow} \dfrac{2}{x} = y \Rightarrow x = \dfrac{2}{y} \Rightarrow x \neq 0 \Rightarrow y \neq 0$

$x \neq -1$

① $\Rightarrow y \neq 0$ ✓ / ② $\rightarrow \underbrace{\dfrac{2}{y}}_{\neq 0 \,✓} \neq -1 \rightarrow \underline{y \neq -2} \Rightarrow R - \{0, -2\} \Rightarrow \underline{-2+0=-2}$

۹) $y = \dfrac{(\sqrt{x}-1)(\sqrt{x}-3)}{(\sqrt{x}-1)(\sqrt{x}-1)} \Rightarrow \dfrac{\sqrt{x}-3}{\sqrt{x}-1} = y \Rightarrow \sqrt{x} = -\dfrac{3-y}{y-1} = \dfrac{y-3}{y-1} \Rightarrow \geq 0$

$(\sqrt{x}-1) \neq 0$

	1		3	
+		−		+

$\dfrac{y-3}{y-1} \neq 1 \rightarrow y-3 \neq y-1$ ✓ $\Rightarrow R_f = (-\infty, 1) \cup [3, +\infty)$

۱۰) $\dfrac{x^2(x+2)-(x+2)}{x^2-1} \Rightarrow \dfrac{(x+2)(x^2-1)}{x^2-1} \Rightarrow x^2-1 \neq 0 \Rightarrow x+2=y \Rightarrow x = \underline{y-2} \rightarrow y-2 \neq 1 \rightarrow y \neq 3$

$y-2 \neq -1 \rightarrow y \neq 1$

$\boxed{1+3=4}$