(۱ الف) $y=(x-2[\frac{x}{2}])^2 \xrightarrow{2\times} (\frac{x}{2}-[\frac{x}{2}])' \longrightarrow 0\le \frac{x}{2}-[\frac{x}{2}]<1$

$\times 2$: $0\le \frac{x}{2}-[\frac{x}{2}] < 2 \xrightarrow{\text{توان ۲}} 0\le \frac{x}{2}-[\frac{x}{2}]<4 \qquad R_f=[0,4)$

ب) $\log y-\sin x-4 \qquad \log y=\sin x+4 \longrightarrow -1\le \sin x\le 1 \to 3\le \sin x+4\le 5$

$\log y$

(۲ $f(x)=\sqrt{\frac{x^2-4}{x^2-9}} \qquad y^2=\frac{x^2-4}{x^2-9} \Rightarrow y^2x^2-9y^2=x^2-4 \Rightarrow y^2x^2-x^2=9y^2-4$

$x^2(y^2-1)=9y^2-4 \to x=\sqrt{\frac{9y^2-4}{y^2-1}} \Rightarrow \frac{9y^2-4}{y^2-1}\ge 0$

	-1		$-\frac{2}{3}$		$\frac{2}{3}$		1	
+		−		+		−		+

$(-\infty,-1)\cup[-\frac{2}{3},\frac{2}{3}]\cup(1,+\infty) \xrightarrow{\text{جذر}} [0,\sqrt{\frac{2}{3}}]\cup(1,+\infty)$

(۳ الف) $x+\frac{1}{x}=t \longrightarrow t+2\sqrt{t} \qquad t\ge 0 \qquad x\ne 0 \Rightarrow t>0$

$x+\frac{1}{x}>0 \longrightarrow x+\frac{1}{x}\ge 2$

$\min \xrightarrow[x=1]{} 2+2\sqrt{2} \qquad R_f=[2+2\sqrt{2},+\infty)$

ب) $y=(3-\sin x)(1+\sin x)=3+3\sin x-\sin x-\sin^2 x$

$\Rightarrow y=-\sin^2 x+2\sin x+3$
- $\sin x=1 \Rightarrow 4$
- $\sin x=-1 \Rightarrow 0 \qquad R_f=[0,4)$
- $\sin x=1 \Rightarrow 4$

$\frac{-b}{2a}$

(۴ $D_f=\{1,2,\dots,8,9\} \qquad x=1 \to y=\frac{-1}{3}+10=\frac{29}{3} \qquad x=2\to y=\frac{28}{3}$

$x=9 \to y=-3+10=7 \qquad x=8\to y=\frac{22}{3}$

$R_f=\{7,\frac{22}{3},\frac{23}{3},8,\frac{25}{3},\frac{26}{3},9,\frac{28}{3},\frac{29}{3}\} \Rightarrow$ جمع $=75$

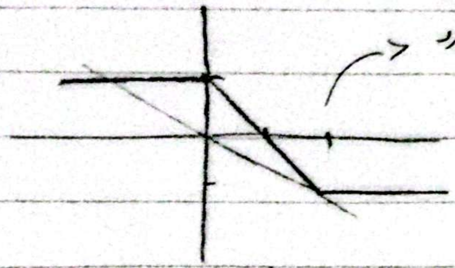
(۵ $\sqrt{ax^2+bx+c} \qquad a=0 \longrightarrow bx+c \qquad \sqrt{x^2+4} \longrightarrow R=[4,+\infty)$ جذر

$\frac{2}{-\ |\ +} \qquad \frac{x-2}{?} \qquad b=1,\ c=-2$

$[2,+\infty)$

6)

$|x-2|-|x-1| = kx$



$y = kx$

$x=2,\ y=-1 \Rightarrow -1 = 2k$

$k = -\frac{1}{2}$

7)

الف) $= x[x]$

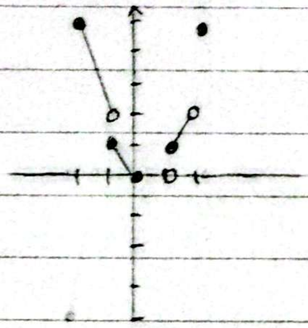
$-2 \le x < -1 \quad -2x$

$-1 \le x < 0 \quad -x$

$0 \le x < 1 \quad 0$

$1 \le x < 2 \quad x$

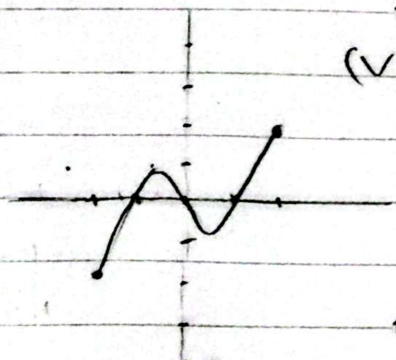
$x = 2 \quad 2x$



$\Rightarrow$ برد: $[0,4]-\{2\}$

ب) $x|x|-x$

$\begin{cases} x^2-x & x \ge 0 \\ -x^2-x & x < 0 \end{cases}$



برد: $[-2,2]$

8)

$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} \Rightarrow \frac{x+1+x+1}{x^2+x} = \frac{2+2x}{x^2+x} \quad x \ne 0,-1 \quad \frac{2(1+x)}{x(1+x)}$

$y = \frac{2}{x}$ برد: $R-\{0,-2\} \Rightarrow a-b \Rightarrow 0+(-2) = -2$)

9)

$y = \frac{x-4\sqrt{x}+3}{x-2\sqrt{x}+1} = \frac{(\sqrt{x}-1)(\sqrt{x}-3)}{(\sqrt{x}-1)^2} \Rightarrow y = \frac{\sqrt{x}-3}{\sqrt{x}-1} \Rightarrow y\sqrt{x}-y = \sqrt{x}-3$

$(y-1)\sqrt{x} = y-3 \Rightarrow x = \sqrt{\frac{y-3}{y-1}} \quad \frac{y-3}{y-1} \ge 0 \quad \begin{matrix} 1 & 3 \\ + \ 0 & - \ 0 \ + \end{matrix} \quad R_f: (-\infty,-1)\cup[3,+\infty)$

10)

$x^3+2x^2-x-2 \ \big| \ x-1$

$-x^3 + x^2 \qquad x^2+3x+2$

$3x^2-x-2$

$-3x^2+3x$

$2x-2$

$-(2x-2)$

0

$y = \frac{(x-1)(x^2+3x+2)}{x^2-1} = \frac{(x-1)(x+2)(x+1)}{(x+1)(x-1)}$

$x \ne \pm 1 \Rightarrow y \ne 1,3 \Rightarrow R: R-\{3,1\}$

$1+3 = 4$)