(۱) الف)

$y = \left(2\left(\frac{x}{2} - \left[\frac{x}{2}\right]\right)\right)^2$

$= 4\left(\frac{x}{2} - \left[\frac{x}{2}\right]\right)^2$

$0 \le \frac{x}{2} - \left[\frac{x}{2}\right] < 1$

$\Rightarrow 0 \le 4\left(\frac{x}{2} - \left[\frac{x}{2}\right]\right)^2 < 4$

$R = [0, 4)$

ب)

$y = -\sin^2 x + 4\sin x - 3$

$1 \to -1 + 4 - 3 = 0$

$-1 \to -1 - 4 - 3 = -8$

$R = [-8, 0]$

(۲)

$g(x) = \frac{x^2 - 4}{x^2 - 9}$

$\xrightarrow{x = \infty} 1$ (مجانب افقی)

$\xrightarrow{x = 0} \frac{4}{9}$

$R = (-\infty, \frac{4}{9}] \cup (1, \infty)$

$\Rightarrow R_{f(x)} = [0, \frac{2}{3}] \cup (1, \infty)$

$0 + \frac{2}{3} + 1 = \frac{5}{3}$

(۳) الف)

$x + \frac{1}{x} \ge 2 \Rightarrow x > 0$

$y = \left(x + \frac{1}{x} + 1\right)^2 - 1$

$2 \le x + \frac{1}{x} < \infty$

$\Rightarrow 3 \le x + \frac{1}{x} + 1 < \infty$

$\Rightarrow 8 \le \left(x + \frac{1}{x} + 1\right)^2 - 1 < \infty$

$\Rightarrow R = [8, \infty)$

(۴)

$\xrightarrow{x=1} 10 - \frac{1}{4} = \frac{39}{4}$

$\xrightarrow{x=9} 10 - \frac{10}{4} = \frac{30}{4}$

$S = \frac{9}{2}\left(\frac{39}{4} + \frac{30}{4}\right) = ۷۷,۵$

(۵)

$a = 0$

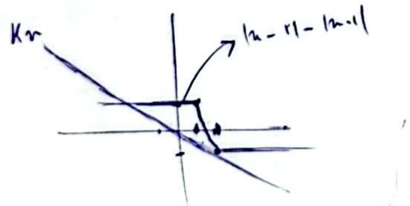
$bx + c = b(x + \frac{c}{b}) = b(x - 2)$

$\xrightarrow{x=2} b = 1 \Rightarrow c = -2$

$g(x) = \sqrt{x^2 + 4}$

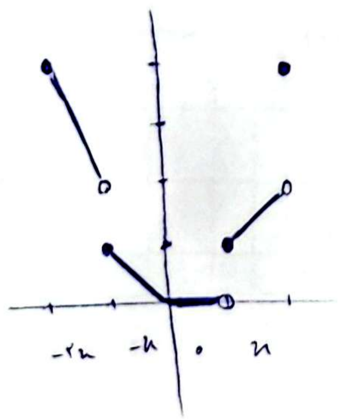
$4 \le x^2 + 4 \Rightarrow R = [2, \infty)$

(۶)



$\xrightarrow{x=1} 2K = -1 \Rightarrow K = -\frac{1}{2}$

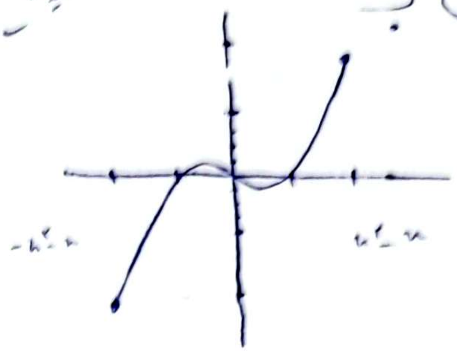
(۷) الف)



$R = [0, 4] - \{2\}$

۷) ب

برای بخش ارزش



$R=[-2,2]$

۸)

$$\frac{(x+1)+x+1}{x^2+x}$$

$$=\frac{2x+2}{x^2+x}=\frac{2(x+1)}{x(x+1)}$$

$$=\frac{2}{x}\xrightarrow{x=-1}-2$$

$$\Rightarrow R=R-\{-2,0\}$$

$$-2+0=-2$$

۹)

$$f(x)=\frac{(\sqrt{x}-1)(\sqrt{x}-3)}{(\sqrt{x}-1)^2}$$

$$=\frac{\sqrt{x}-3}{\sqrt{x}-1}$$

$$\xrightarrow{\sqrt{x}=0}3$$

$$\xrightarrow{\sqrt{x}=\infty}1$$

$$R=(-\infty,1)\cup[3,\infty)$$

۱۰)

$$f(x)=\frac{x(x^2-1)+2(x^2-1)}{x^2-1}$$

$$=\frac{(x^2-1)(x+2)}{x^2-1}=x+2$$

$$\xrightarrow{x=1}3$$

$$\xrightarrow{x=-1}1$$

$$R=R-\{1,3\}$$

$$1+3=4$$

۱) ب

$$\log y=\sin x+4$$

$$y=10^{\sin x+4}$$

$$3\leq \sin x+4\leq 5$$

$$\Rightarrow 10^3\leq 10^{\sin x+4}\leq 10^5$$

$$\Rightarrow R=[1000,100000]$$