محمد رضا

① الف)

$D_{f(x)} = [-4, 2]$

$D_{f(3x-2)} = \rightarrow 3x-2=-4 \Rightarrow 3x=-2 \rightarrow x=\frac{-2}{3}$

$3x-2=2 \Rightarrow 3x=4 \Rightarrow x=\frac{4}{3}$

$D_{f(3x-2)} = [\frac{-2}{3}, \frac{4}{3}]$

ب)

$D_{f(3x-2)} = [-4, -2]$

$D_{f(x)} \Rightarrow 3x-2 \xrightarrow{x=-4} -12-2=-14 \quad 3x-2 \xrightarrow{x=2} 6-2=4$

$D_{f(x)} = [-14, 4]$

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②

$f(x) = x + \sqrt{x-2} \rightarrow f(x-2) = (x-2) + \sqrt{(x-2)-2}$

$\Rightarrow x-2 + \sqrt{(x-2)-2} \Rightarrow y = x-2+\sqrt{x-4} \rightarrow \sqrt{x-4}=t$

$y = t^2+t+2 \Rightarrow t^2+t+2 = x \xrightarrow{x=t^2+4}$

$\cancel{t^2}+t+2 = \cancel{t^2}+4 \Rightarrow t=2 \Rightarrow \sqrt{x-4}=2 \Rightarrow x=8$

$(8,8) \rightarrow$ نقطه‌ای که قطع می‌کند

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③

$2f(3x+1)$
- $\rightarrow (0,0) \rightarrow f(1)=0$
- $\rightarrow (3,4) \rightarrow 2f(10)=4 \rightarrow f(10)=2$
- $\rightarrow (4,0) \rightarrow 2f(13)=0 \rightarrow f(13)=0$

$f(x) = \left.\begin{matrix}(1,0)\\(10,2)\\(13,0)\end{matrix}\right\} \xrightarrow[\text{مثلث}]{\text{مساحت}} \frac{12\times 2}{2} = 12$

$f(x) = (2,-3)$ ④

نقطه‌متناظر $\rightarrow y = a + 2f\left(\dfrac{x'-a}{2}\right) \Rightarrow 2 = \dfrac{x'-a}{2} \Rightarrow$

$4 = x'-a \Rightarrow x' = 4+a$

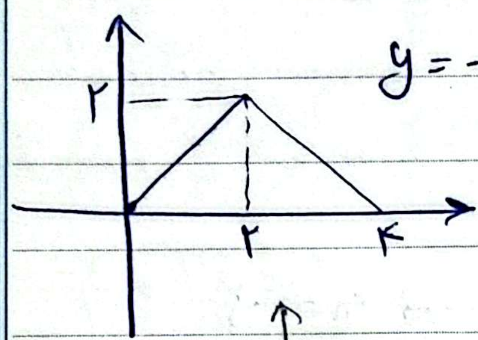
$y' = a + 2\underbrace{f(2)}_{-3} \Rightarrow y' = a-6$

$y' = 2x'-8 \rightarrow a-6 = 2(4+a)-8 \Rightarrow a-6 = 8+2a-8$

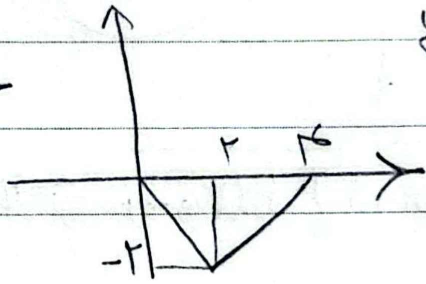
$\Rightarrow \boxed{a=-6}$

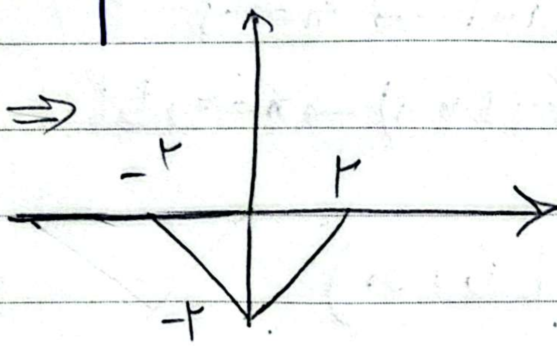
⑤

$y = -f(x-2)$



$y = f(x-2)$

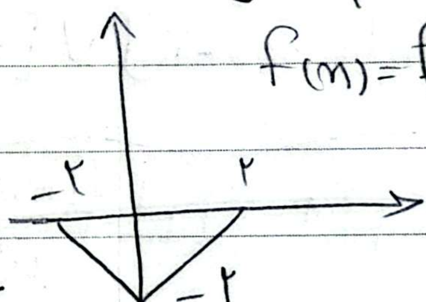




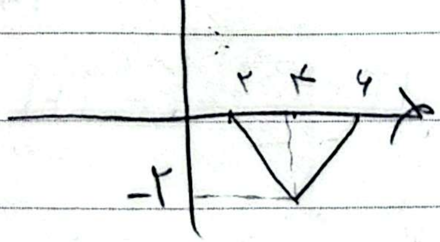
$y = f(x)$

$y = \dfrac{1}{2} f(4-x)$

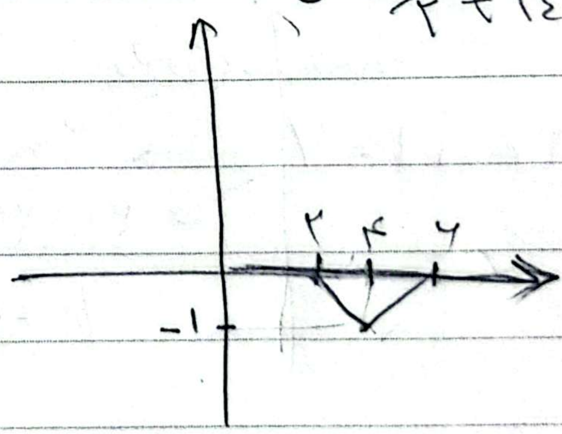
$f(x) = f(-x) = y$



$f(4-x)$

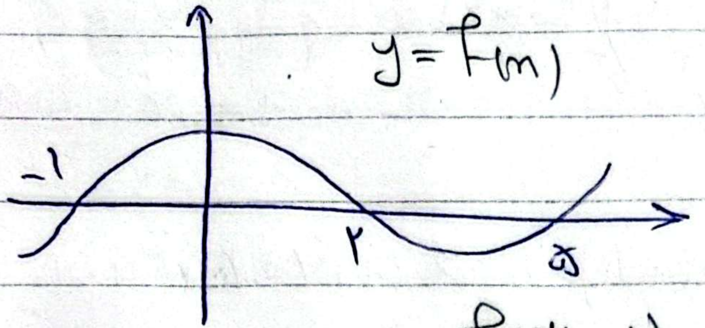


$y = \dfrac{1}{2} f(4-x)$

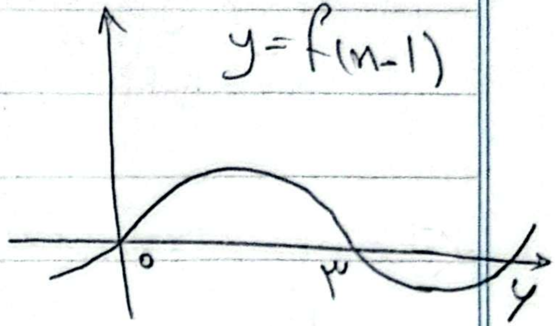


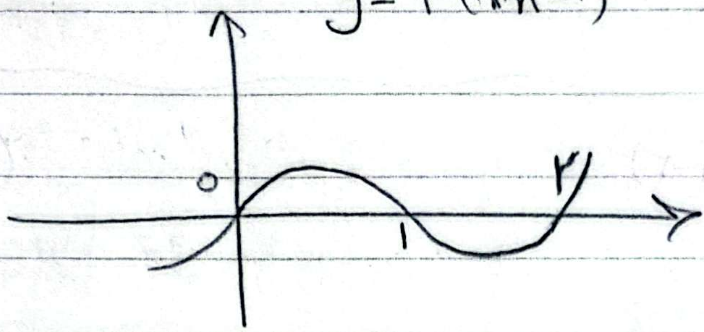
۶

$y=\sqrt{-(x+۲)f(۳x-۱)}$



$\Rightarrow$





$y=\sqrt{-(x+۲)f(۳x-۱)} \Rightarrow -x-۲ \rightarrow x=-۲$

$f(۳x-۱) \rightarrow x=۰,۱,۲$

-۲		۰		۱		۲	
$+$	$-$		$+$		$-$		$+$

$D_f=(-\infty,-۲]\cup[۰,۱]\cup[۲,+\infty)$

۷

$f(x)=|۲x-۳|+۱ \xrightarrow[k\text{ تا چپ}]{۳\text{ واحد پایین}} f(x+k)=|۲x+۲k-۳|-۲$

$\underbrace{|۲x+۲k-۳|-۲=|۲x-۳|+۱}_{[illegible]\ x=۰} \Rightarrow |۲k-۳|-۲=۳+۱$

$|۲k-۳|=۶ \longrightarrow ۲k-۳=۶ \longrightarrow ۲k=۹ \longrightarrow k=۴{,}۵$

$\searrow ۲k-۳=-۶ \longrightarrow ۲k=-۳ \longrightarrow k=-۱{,}۵$ غ ق ق

صورت سئوال گفته . (k مثبت) $k=۴{,}۵$

$$f(x)=\frac{2x-1}{x+1}\longrightarrow\frac{-2x+1}{x+1}\longrightarrow 3f(x)\rightarrow$$ ۸

$$3\left(\frac{-2x+1}{x+1}\right)\Rightarrow\frac{-6x+3}{x+1}+k=\frac{2x-1}{x+1}\Rightarrow$$

$$\frac{-6x+3+kx+k}{x+1}=\frac{2x-1}{x+1}\Rightarrow\frac{(-6+k)x+3+k}{x+1}=\frac{2x-1}{x+1}$$

$-3+2k$

$$\Rightarrow(-6+k)x^2+3x+kx+(-6+k)x+3+k=2x^2+x-1$$

$$(k-8)x^2+(2k-4)x+k+4\rightarrow$$ تقاطع ندارند پس منفی

$k^2-4k-32$

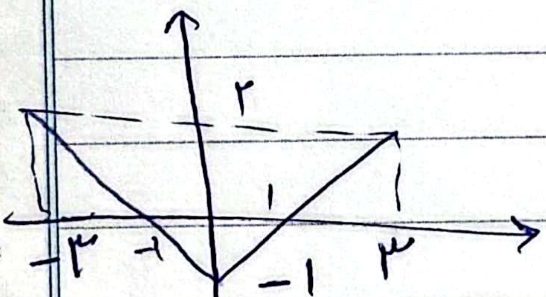
$$(2k-4)^2-4(k+4)(k-8)=0$$

$$\cancel{4k^2}+16-\cancel{16k}-\cancel{4k^2}+\cancel{16k}+128$$

۴

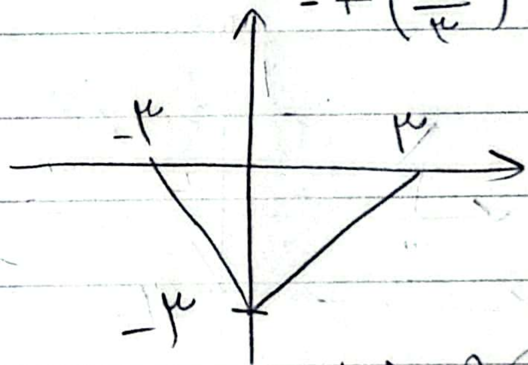
$$y_1=-f\left(\frac{x}{3}\right)+2$$

$$y_2=f(x-1)$$

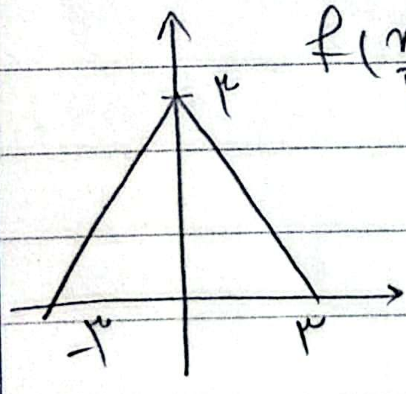


$\rightarrow$

$-f\left(\frac{x}{3}\right)$

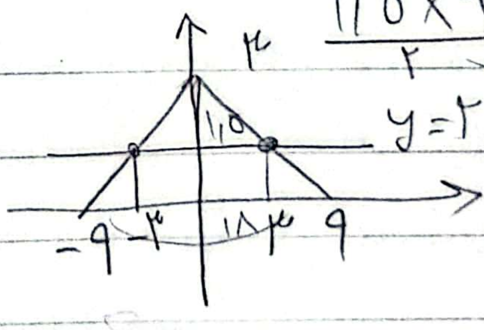


$f\left(\frac{x}{3}\right)$



$\rightarrow$

$\frac{1.5\times 9}{2}=6.75$



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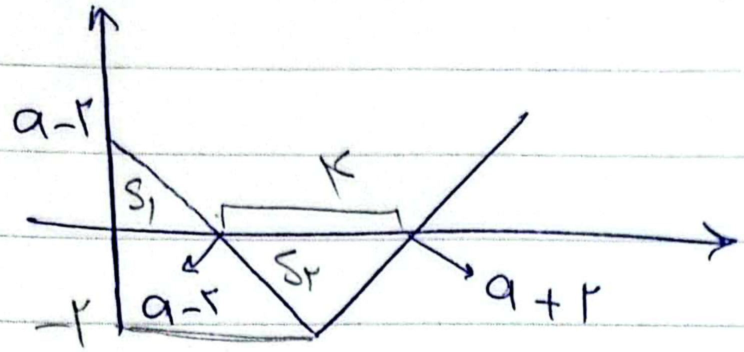

$\frac{۴x+۲}{۲} - ع$

$S_1 + S_۲ = \frac{۹}{۲}$

$S_1 = \frac{۱}{۲}$

$\frac{(a-۲)(a-۲)}{۲} = \frac{۱}{۲} \rightarrow (a-۲)^۲ = ۱ \rightarrow a = ۳ \checkmark$

$a = ۱$ غ ق ق

$f(x) = |x - ۳| - ۲$

$f(\frac{۱}{۳}) \rightarrow |\frac{۱}{۳} - ۳| - ۲ \rightarrow \frac{۸}{۹} - ۲ \rightarrow \frac{-۱۰}{۹}$