نام: رسا احمدی   شماره شیت: ۹   مقطع: دوازدهم ریاضی B

①

$f(t) = \begin{cases} 3t-1 & t \geq 3 \\ 4t-9 & t < 3 \end{cases} \rightarrow f(3-x) = \begin{cases} 8-3x & x \leq 0 \\ 3-4x & x > 0 \end{cases} \qquad f(x+1) = \begin{cases} 3x+2 & x \geq 2 \\ 4x-5 & x < 2 \end{cases}$

$t = x+3$

$D \geq 0 \begin{cases} x \leq 0 \rightarrow D = 8-3x-(4x-5) = 13-7x \rightarrow 13-7x \geq 0 \rightarrow x \leq \frac{13}{7} \xrightarrow{x \leq 0} x \in (-\infty, 0] \quad ① \\ 0 < x < 2 \rightarrow D = 3-4x-(4x-5) = 8-8x \rightarrow 8-8x \geq 0 \rightarrow x \leq 1 \xrightarrow{0<x<2} x \in (0, 1] \quad ② \\ x \geq 2 \rightarrow D = 3-4x-(3x+2) = 1-7x \rightarrow 1-7x \geq 0 \rightarrow x \leq \frac{1}{7} \xrightarrow{x \geq 2} x \in \phi \quad ③ \end{cases}$

$① \cup ② \cup ③ \longrightarrow x \in (-\infty, 1]$

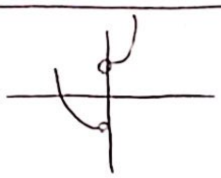
②

$f(x) = (x^3+x)^3 \rightarrow$ اکیداً صعودی $\rightarrow f \circ f(x) < f(x^3) \rightarrow f(x) < x^3 \rightarrow (x^3+x)^3 < x^3 \rightarrow x^3 < 0$

$\begin{array}{c|c} & 0 \\ - & + \end{array} \rightarrow x \in (-\infty, 0)$

③

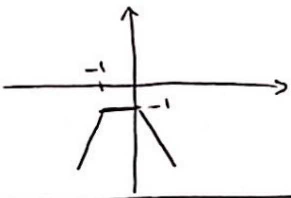
$-\frac{x(x^2+1)}{x} = -x^2-1 \quad \Big|_{0} \quad \frac{x(x^2+1)}{x} = x^2+1$



$\rightarrow$ نایکنوا

④

$\frac{2x+1}{|x|-|x+1|} \times \frac{|x|+|x+1|}{|x|+|x+1|} = \frac{(2x+1)(|x|+|x+1|)}{x^2-(x+1)^2} = -|x|-|x+1| \qquad \begin{array}{c|c|c} -1 & & 0 \\ 2x+1 & -1 & -2x-1 \end{array}$



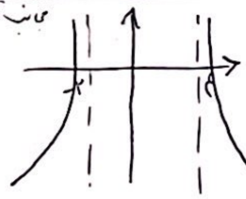
بازه صعودی $= (-\infty, 0] \rightarrow \boxed{a = 0}$

⑤

$\log_{\frac{1}{2}} x^2-2x-8 = \log_{\frac{1}{2}} (x-4)(x+2) \rightarrow \begin{array}{ccccc} & -2 & & 4 & \\ + & | & - & | & + \end{array}$

تعریف شده

تعریف نشده



$\rightarrow (-\infty, -2)$ در بازه اکیداً صعودی است

⑥

الف) $a^2-3 > 1 \rightarrow a^2 > 4 \rightarrow |a| > 2 \rightarrow a > 2 \cup a < -2$

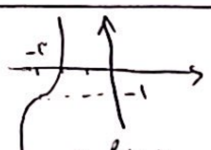
ب) $a^2-3 \geq 1 \rightarrow a^2 \geq 4 \rightarrow |a| \geq 2 \rightarrow a \geq 2 \cup a \leq -2$

⑦

$f(-2) = -1$

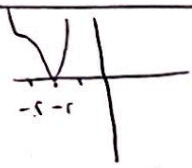
$f(x) = x^3+ax^2+bx+c$

$\rightarrow f(x) = (x+2)^3-1 \Rightarrow$



$y = f(x)$

$\rightarrow$



$y = |f(x)|$

$\rightarrow$ اکیداً صعودی $= (k, +\infty) = (-2, +\infty) \rightarrow \boxed{K = -2}$

۸)

$f(f(x)) \le f(\sqrt{x}) \to f(x) \le \sqrt{x} \to x+\sqrt{x}-2 \le \sqrt{x} \to x \le 2$ ①

$\sqrt{x} \Rightarrow x \ge 0$ ② $\to$ ①$\cap$② $\Rightarrow x \in [0,2] \to$ سه عدد صحیح

۹)

$9\cos^2 x - 1 \ge 0 \to \cos^2 x \ge \frac{1}{9} \to 1 \ge \cos \ge \frac{1}{3} \xrightarrow[t \in [-1,8]^*]{t = 9\cos^2 x - 1} 2^{\sqrt[3]{t}} - 2^{\sqrt[3]{-t}} = 2^{\sqrt[3]{t}} - 2^{-\sqrt[3]{t}}$

$\xrightarrow{*} A \in [-1,2] \Rightarrow \begin{cases} f(-1) = -\frac{3}{2} \\ f(2) = \frac{15}{4} \end{cases} \to$ صعودی است $\to [a,b] = [-\frac{3}{2}, \frac{15}{4}] \to b-a = \frac{21}{4}$

۱۰)

$f(x) = x - [x+2] = x - [x] - 2 \qquad g(x) = \frac{2^{x} - 2^{-x}}{2} = 2^{x-1} - 2^{-x-1}$

$g \circ f(x) = 2^{x-[x]-2-1} - 2^{-x+[x]+2-1} = 2^{x-[x]-3} - 2^{[x]-x+1}$

$\xrightarrow{0 \le x-[x] < 1} \left.\begin{matrix} \min = 2^{-3} - 2^{1} = -\frac{15}{8} \\ \max = 2^{-2} - 2^{0} = -\frac{3}{4} \end{matrix}\right\} \to R_{g \circ f} = [-\frac{15}{8}, -\frac{3}{4}]$